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IACPE **No 19, Jalan Bilal Mahmood** **80100 Johor Bahru** **Malaysia**	**Practical Mathematics** **CERTIFIED PROCESS TECHNICAN TRAINING MODULE**	**Great Job with a Great Future**

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3. **Encourage practicing engineers and management professionals to participate in a continuing program of personal and professional development**

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INTRODUCTION

Scope

This module introduces and then consolidates basic mathematical principles and promotes awareness of mathematical concepts for students needing a broad base for further vocational studies. It provides a foundation in mathematical principles, which will enable students to solve mathematical, scientific and associated technical principles. The material will provide applications and mathematical principles necessary for advancement onto a range of technical profiles. It is widely recognized that a students' ability to use mathematics is a key element in determining subsequent success.

There are tables and figures that assist in making these math calculations from the various reference sources. All the important parameters use in the guideline are explained in the definition section which help the reader more understand the meaning of the parameters or the term used.

In theory section, we will explain about fractions, decimals, percentage, notation area, volume, measurment, unit conversion, and statistics. Theory is introduced by a brief outline of essential theory, definitions, formulae, laws and procedures.

General Design Consideration

In many workplace settings, workers need to know how to do basic computation problems involving decimals, percents, and fractions. Sometimes it may also know how to do word problems. These mathematical skills are designed to develop the knowledge and understanding the basic math and geometry.

Mathematics plays a critical role in the daily operations of a chemical plant. Every day operators need to adjust batch formulations, reconcile inventory, and calculate production volumes and production schedules. Each of these activities requires a firm grasp on the principles of mathematics.

Specific skills a chemical plant operator must hone include the basic ability to add, subtract, multiply, and compute fractions and percentages. This module covers these skills by reviewing key math principles while using plant examples.

<u>Steps to Problem Solving</u>

1) Define the problem

- What am I being asked to do or find?
- What information have I been given?
- Is there other information that I need to know or need to find?
- Will a sketch help?
- Can I restate the problem in my own words?
- Are there any key words?

2) Decide on a plan

- What operations do I need to perform and in what order?
- On which numbers do I perform these operations?

3) Carry out the plan

4) Examine the outcome

- Is this a reasonable outcome?
- Does the outcome make sense in the original problem?

- If I estimated the answer would it be close to the result?
- Does the outcome fall outside any limits in the problem? (too large or too small)

Another way to determine what operation needs to solve word problems is to use the given information. If the given information includes a total value, the operation is most likely subtraction or division. If the problem asks for a total, the operation is always addition or multiplication. Multiplication is a shortcut for addition and should be used when the numbers being totaled are the same.

Addition

- added to	- in all
- additional	- increase of
- all together	- increased by
- combined	- more than
- gain of	- plus
- how many all together	- sum
- how many in all	- total
- how much all together	

Subtraction

- how many more	- dropped
- how many less	- have left
- how many left	- less
- how many fewer	- less than
- how many remain	- loss of
- how much more	- minus
- how much less	- remaining
- change	- save
- decrease	- take away
- decreased by	- difference

Multiplication

- double	- triple
- product	- twice
- times	- twice as much

- how many in all (with equal numbers)
- how much (with equal amounts)
- of (with fractions and percents)

- total (of equal numbers)

Division

- divided by - goes into
- divided equally - quotient
- divided into - what's half
- evenly - how many per
- how many in each

Addition/Subtraction of Fractions

Frequently a mathematical expression appears as a fraction with one or more fractions in the numerator and/or the denominator. To simplify the expression multiply the top and bottom of the fraction by the reciprocal of the denominator.

Simplifying fractions by addition or subtraction requires the use of the lowest common denominator. The denominator on both fractions must be the same before performing an operation. Multiplication with fractions is very straightforward, just multiply numerator by numerator and denominator by denominator. When dividing with a fraction, the number being divided (dividend) is multiplied by the reciprocal of the divisor. Frequently this has been stated "invert and multiply."

Converting

One process to convert within the English system of measurement is to determine if the measure start with is larger or smaller than the measure you are trying to convert to. Follow these two rules to use this process:

To Convert from a Larger Unit to a Smaller Unit:

- Refer to an equivalency table to find a relationship using both quantities
- Multiply
- Add if necessary

To Convert from a Smaller Unit to a Larger Unit:

- Refer to an equivalency table to find a relationship using both quantities
- Divide
- Express any remainder as an equivalent, smaller unit

Charts and graphs

In this module will look at three different graphics: bar graphs, circlegraphs, and line graphs. Bar graphs give information on the left and at the bottom. Circle or pie graphs are also easy to read. A circle graph is divided into parts (often percentages). The percents must add up to 100%. Line graphs are similar to bar graphs. You read the data at the point where the bottom information intersects with the side information.

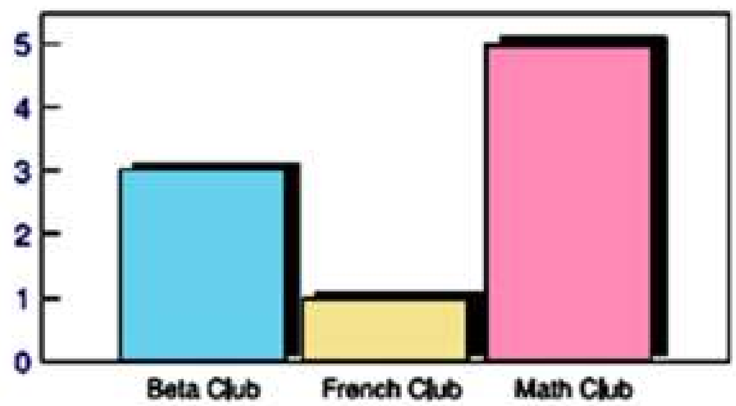


Figure 1: Bar graph

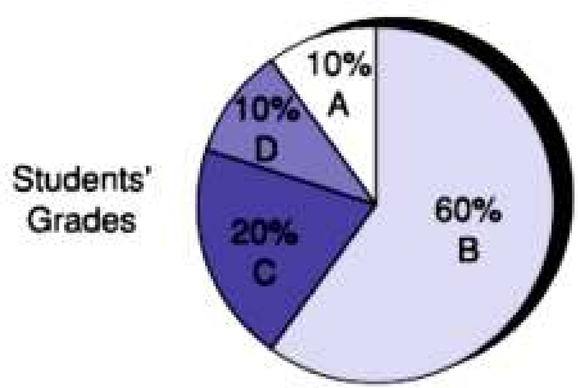


Figure 2: Circle graph

Figure 3: Line graph

DEFINITIONS

Convert - to change to another form

Deductions – subtractions

Decimals - represent values less than 1

Denominator - The number on bottom of the fraction

Fraction - a part of any quantity, object, or number

Frequency - the number, proportion, or percentage of items in a particular category in a set of data

Mensuration - a branch of mathematics concerned with the determination of lengths, areas and volumes.

Numerator - The number on top of the fraction

Planimeter - an instrument for directly measuring small areas bounded by an irregular curve

Quadrilateral - a polygon with four edges (or sides) and four vertices or corners. Sometimes, the term quadrangle is used, by analogy with triangle, and sometimes tetragon for consistency with pentagon (5-sided), hexagon (6-sided) and so on.

Rate - a ratio or comparison of 2 different kinds of measures

Ratio - a comparison of 2 numbers expressed as a fraction, in colon form, or with the word “to”

Statistics - a branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data

Surface area - the sum of all the areas of all the shapes that cover the surface of the object

Theorem - a formula, proposition, or statement in mathematics or logic deduced or to be deduced from other formulas or propositions

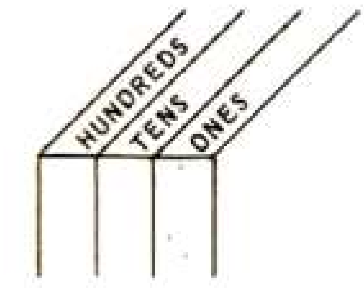
NOMENCLATURE

%	percentage
Ω	electrical resistance, ohm
V	volume
A	area
l	length
b	width
h	height
d	diameter
r	radius
A_S	surface area
A_{base}	base area
A_{curved}	Curved area
$\bar{x}$	mean value
f	frequency
σ	standard deviation
Q	quartiles
x	a member of the set
n	the number of members in the set.
π	circle constant 3.14 or 22/7

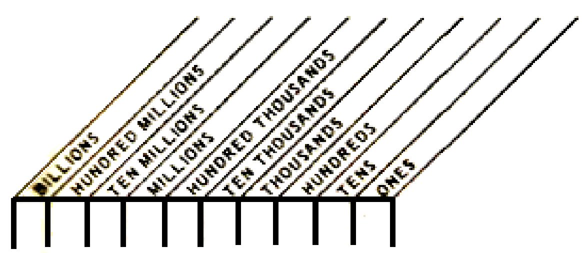
THEORY

The Numbers of Counting and Operations

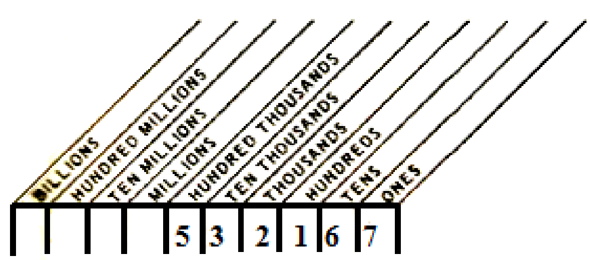
The digits make up the numerals that use to represent the “number” of things 0, 1, 2, 3, 4, 5 ,6, 7, 8, and 9



For the number larger than three digits (over 999) are usually marked off with common in thousands and million reading.



Example: read the number



It say: five hundred - thirty two thousand, one hundred and sixty-seven.

The four operations of arithmetic are addition, substraction, multiplication, and division.

Addition is finding the total, or sum, by combining two or more numbers.

Example: $5 + 11 + 3 = 19$ is an addition

Subtraction is taking one number away from another.

Example: $5 - 2 = 3$

The basic idea of multiplication is repeated addition.

For example: $5 \times 3 = 5 + 5 + 5 = 15$

But as well as multiplying by whole numbers, we can also multiply by fractions, decimals and more.

For example $5 \times 3\frac{1}{2} = 5 + 5 + 5 +$ (half of 5) $= 17.5$

Division is splitting into equal parts or groups. It is the result of "fair sharing".

Example: there are 12 chocolates, and 3 friends want to share them, how do they divide the chocolates?

Answer: They should get 4 each.

It use the ÷ symbol, or sometimes the / symbol to mean divide:

$12 \div 3 = 4$ or

$12 / 3 = 4$

Estimation

Estimation is finding a number that is close enough to the right answer. It can say approximately (≈). Estimation can save time (when the calculation does not have to be exact)

Example: you want to plant a row of flowers. The row is 58.3cm long. The plants should be 6cm apart. How many do you need?

"58.3 is nearly 60, and 60 divided by 6 is 10, so 10 plants should be enough."

One very simple form of estimation is rounding. Rounding is often the key skill need to quickly estimate a number. This is where you make a long number simpler by 'rounding', or expressing in terms of the nearest unit, ten, hundred, tenth, or a certain number of decimal places.

For example, 1,654 to the nearest thousand is 2,000. To the nearest 100 it is 1,700. To the nearest ten it is 1,650.

The way it works is simple: you simply look at the number on the right of the level that you are rounding to and see whether it is closer to 0 or 10.

In practice, this means that if you've been asked to round to the nearest 10, you look at the units, if to three decimal places, you look at the fourth number to the right of the decimal point and so on. If that number is 5 or over, you round up to the next number, and if it is 4 or under, you round down.

Example: Express 0.4563948 to three decimal places.

As you're working to three decimal places, the answer will start 0.45.

To work out whether the next number is 6 or 7, you need to look at the fourth number, which is 3. As 3 is less than 5, you round down.

The answer therefore is 0.456.

Equal, Greater or Less Than

As well as the familiar equals sign (=) it is also very useful to show if something is not equal to (≠) greater than (>) or less than (<).

<u>Less Than and Greater Than</u>

To remember which way around the "<" and ">" signs go, just remember:

BIG > small

small < BIG

Example:

10 > 5

"10 is greater than 5"

Or the other way around:

5 < 10

"5 is less than 10"

... Or Equal To ...

Sometimes we know a value is smaller, but may also be equal to! To show this, we add an extra line at the bottom of the "less than" or "greater than" symbol like this:

The "less than or equal to" sign: ≤

The "greater than or equal to" sign: ≥

These are the important signs to know:

Tabel 1: Sign of equal, greater, and less

=	When two values are equal we use the "equals" sign	example: **2+2 = 4**
≠	When two values are definitely **not** equal we use the "not equal to" sign	example: **2+2 ≠ 9**
<	When one value is smaller than another we use a "less than" sign	example: **3 < 5**
>	When one value is bigger than another we use a "greater than" sign	example: **9 > 6**
≥	greater than or equal to	marbles ≥ 1
≤	less than or equal to	dogs ≤ 3

Combining

We can sometimes say two (or more) things on the one line

Example: Sam cuts a 10m rope into two. How long is the one piece?

Let us call the cut of rope "L", so, L must be greater than 0 m (otherwise it isn't a piece of rope), and also less than 10m.

$$L > 0$$

$$L < 10$$

That says that L (the cut of rope) is between 0 and 10 (but not 0 or 10)

$$0 < L < 10$$

Order Of Operations

A universal agreement exists regarding the order in which addition, subtraction, multiplication, and division should be performed.

1. Powers and roots should be performed first.
2. Multiplication and division are performed next from left to right in the order that they appear.
3. Additions and subtractions are performed last from left to right in the order that they appear.

Example 1: $3 + 4 \times 5 =$
$3 + 20 = 23$

Example 2: $7 \times 3^2 =$
$7 \times 9 = 63$

Every negative number has its positive counterpart, which is sometimes called its additive inverse. The additive inverse of a number is that number which when added to it produces 0. Thus, the additive inverse of -5 is +5 because (-5) + (+5) = 0.

Subtracting a negative number is the same as adding its positive counterpart. Adding a negative number is the same as subtracting its positive counterpart.

Example: 12 + (–8) – (–10) =

12 + (–8) + (10) =

12 – (8) + (10) = 14

When numbers of opposite signs are multiplied or divided, the result is negative. When numbers of the same sign are multiplied or divided, the result is always positive. When dividing or multiplying, the two negative signs cancel out.

Example 1: 6 × (–7) ÷ 3 =

(–42) ÷ 3 = (–14)

Fractions, Decimals and Percentage

<u>Fractions</u>

A fraction is a part of any quantity, object, or number. When 1 is divided by 2, it may be written as 1/2. 1/2 is called a fraction. The number above the line, i.e. 1, is called the "numerator" and the number below the line, i.e. 2 is called the "denominator". When the value of the numerator is less than the value of the denominator, the fraction is called a proper fraction; thus 1/2 is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an improper fraction. Thus 5/2 is an improper fraction and can also be expressed as a mixed number, that is, an integer and a proper fraction. Thus the improper fraction 5/2 is equal to the mixed number 2 ½.

$$\frac{Numerator}{Deno\min ator}$$

Fractions can be added, subtracted, multiplied, or divided. For example in order to figure out the total length of pipe 5/16", 15/16", and ¾". To add these fractions, it must have a common denominator. In this case, the common denominator is 16 because the fraction, ¾" can be expressed as 12/16". Therefore, the sum of the components can be expressed as:

$$\frac{5}{16}'' + \frac{15}{16}'' + \frac{12}{16}'' = \frac{32}{16}'' = 2''$$

To multiply fractions, first multiply the numerator of the first fraction by the numerator of the second fraction, then multiply the denominators. If it is possible, divide the numerator by the denominator to get final answer. For example:

$$\frac{3}{4} \times \frac{2}{9} = \frac{6}{36} = \frac{1}{6}$$

When dividing fractions, the rule is to invert the divisor and multiply. Refer to the example below:

$$\frac{3}{4} \div \frac{2}{9} = \frac{3}{4} \times \frac{9}{2} = \frac{27}{8} = 3\frac{3}{8}$$

Frequently a mathematical expression appears as a fraction with one or more fractions in the numerator and/or the denominator. To simplify the expression multiply the top and bottom of the fraction by the reciprocal of the denominator.
Example:

$$\frac{\frac{2}{5}}{4} =$$

$$\frac{\frac{1}{4} \times \frac{2}{5}}{\frac{1}{4} \times 4} = \frac{\frac{2}{20}}{1} =$$

$$\frac{2}{20} = \frac{1}{10}$$

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called cancelling. Cancelling by 0 is not permissible.

Example:

Find the value of $\frac{3}{7} \times \frac{14}{15}$

Dividing numerator and denominator by 3 gives:

$$\frac{{}^{1}\cancel{3}}{7} \times \frac{14}{\cancel{15}_{5}} = \frac{1}{7} \times \frac{14}{5} = \frac{1 \times 14}{7 \times 5}$$

Dividing numerator and denominator by 7 gives:

$$\frac{1 \times \cancel{14}^{2}}{{}_{1}\cancel{7} \times 5} = \frac{1 \times 2}{1 \times 5} = \frac{2}{5}$$

The order of precedence of operations for problems containing fractions is the same as that for integers, i.e. remembered by BODMAS (Brackets, Of, Division, Multiplication,Addition and Subtraction).

Example:

Simplify: $\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{4 \times 2 + 5 \times 1}{20} \div \frac{\cancel{3}^{1}}{\cancel{24}_{8}} \qquad \text{(B)}$$

$$= \frac{1}{3} - \frac{13}{{}_{5}\cancel{20}} \times \frac{\cancel{8}^{2}}{1} \qquad \text{(D)}$$

$$= \frac{1}{3} - \frac{26}{5} \qquad \text{(M)}$$

$$= \frac{(5 \times 1) - (3 \times 26)}{15} \qquad \text{(S)}$$

$$= \frac{-73}{15} = -4\frac{13}{15}$$

<u>Ratio and proportion</u>

A ratio is a comparison of two values. Ratios are two numbers written in fractional form. The ratio of one quantity to another is a fraction, and is the number of times one quantity is contained in another quantity of the same kind.

Proportions are two equal ratios. You can use proportions to solve for an unknown quantity. If one quantity is directly proportional to another, then as one quantity doubles, the other quantity also doubles. When a quantity is inversely proportional to another, then as one quantity doubles, the other quantity is halved.

For example, a gear wheel having 80 teeth is in mesh with a 25 tooth gear. What is the gear ratio?

Gear ratio = 80 : 25 (divided them with same number, for example 5).
Thus the ratio is 16 : 5

Decimals

Decimals represent values less than 1. The decimal system of numbers is based on the digits 0 to 9. A number such as 53.17 is called a decimal fraction, a decimal point separating the integer part, i.e. 53, from the fractional part, i.e. 0.17. A number which can be expressed exactly as a decimal fraction is called a terminating decimal and those which cannot be expressed exactly as a decimal fraction are called non-terminating decimals. Thus, 3/2 =1.5 is a terminating decimal, but 4/3 =1.33333. . . is a nonterminating decimal. 1.33333. . . can be written as 1.3, called 'one point-three recurring'.

The following examples illustrate how to determine decimal values from fractions.

$$\frac{1}{10} = 0.1 \quad \frac{53}{1{,}000} = 0.053 \quad \frac{27}{100} = 0.27$$

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

(i) correct to a number of significant figures, that is, figures which signify something,

(ii) correct to a number of decimal places, that is, the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4, but is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9.

Thus, the non-terminating decimal 7.6183 becomes 7.62, correct to 3 significant figures, since the next digit on the right is 8, which is in the group of numbers 5, 6, 7, 8 or 9. Also 7.6183. . . becomes 7.618, correct to 3 decimal places, since the next digit on the right is 3, which is in the group of numbers 0, 1, 2, 3 or 4.

Adding and subtracting decimal values is straightforward. Remember to be sure that the decimal places are properly aligned.

$$\begin{array}{r} 0.1136 \\ \underline{+0.0113} \\ 0.1249 \end{array} \qquad \begin{array}{r} 0.1136 \\ \underline{-0.0113} \\ 0.1023 \end{array}$$

The multiplication of decimals is carried out exactly like the multiplication of whole numbers. When the product is obtained, the decimal point must be placed as many places from the right of the product as there are decimal points in the factors.

$$\begin{array}{r} 0.22 \\ \underline{\times 0.04} \\ 0.0088 \end{array} \qquad \begin{array}{r} 0.09 \\ \underline{\times 0.09} \\ 0.0081 \end{array}$$

To divide a decimal by a decimal, move the decimal number of the divisor as many places to the right as necessary to make it a whole number. Next, move the decimal point of the dividend the same number of places to the right, adding zeros as necessary. Place the decimal point in the answer directly above the new decimal point before dividing.

$$0.13 \div 0.077 = 0.077\overline{)0.13} = 77\overline{)130.0} = 1.688$$

The same technique is used to divide a whole number by a decimal.

When dividing a decimal by a whole number, divide as usual placing the decimal point in the answer directly above the decimal point in the number being divided.

$$6.72 \div 17 = 17\overline{)6.72} = 0.395$$

Table 2: Decimal Equivalents of Common Fractions

	1/32	0.03125
1/16	2/32	0.0625
	3/32	0.09375
1/8	4/32	0.125
	5/32	0.15625
3/16	6/32	0.1875
	7/32	0.21875
1/4	8/32	0.25
	9/32	0.28125
5/16	10/32	0.3125
	11/32	0.34375
3/8	12/32	0.375
	13/32	0.40625
7/16	14/32	0.4375
	15/32	0.46875
1/2	16/32	0.50
	17/32	0.53125
9/16	18/32	0.5625
	19/32	0.59375
5/8	20/32	0.625
	21/32	0.65625
11/16	22/32	0.6875
	23/32	0.71875
3/4	24/32	0.75
	25/32	0.78125
13/16	26/32	0.8125
	27/32	0.84375
7/8	28/32	0.875
	29/32	0.90625
15/16	30/32	0.9375

	31/32	0.96875

Percentages

Percent means “per hundred” and is written with the sign, %. Percentages are used to give a common standard and are fractions having the number 100 as their denominators. For example, 25 per cent means 25/100 i.e. ¼ and is written 25%.

For purpose of calculation, percentages are translated into decimal fractions to the hundredths place.A decimal fraction is converted to a percentage by multiplying by 100. Thus,

1.875 corresponds to 1.875 × 100%, i.e. 187.5%

To convert fractions to percentages, they are

(i) converted to decimal fractions and

(ii) multiplied by 100

By division, 5/16 = 0.3125, hence
5/16 corresponds to 0.3125×100%, i.e. 31.25%

Problems involving percent are three kinds, depending on the missing quantity: base, rate or percentage.

Example, 6% of 258 equals 15.48.

6% = rate (always a percent)
258 = base (the original quantity, 100%)
15.48 = percentage (what part of the base)

Or, percentage = base x rate

Example” if you invest $700 at 5¼ % interest annually, what is the percentage of interest your money will earn a year?

percentage = base x rate
P = 700 x 0.0525 (5¼ % = 0.0524)

P = $ 36.75

Notation

Exponents

Exponents were invented to make it easier to write certain expressions involving repetitive multiplication:

K × K × K × K × K × K × K × K × K = K^9.

Note that the exponent (9) specifies the number of times the base (K) is used as a factor rather than the number of times multiplication is performed.

Example: $6^4 = 6 \times 6 \times 6 \times 6 = 1,296$

Fractional Exponents

The definition of a fractional exponent is as follows:

$$X^{M/N} = \sqrt[N]{X^M}$$

This equality converts an expression with a radical sign into an exponent so that the yx key found on most financial calculators can be used.

Example 1: $12^{4/5} = \sqrt[5]{12^4} = 7.3009$

Example 2: $\sqrt[5]{10} = 10^{1/5} = 10^{0.2} = 1.5849$

Subscripts

Concepts or variables that are used in several equations generally use subscripts to differentiate the values. Capitalization rates are expressed as a capital “R.” Since there are a number of different capitalization rates used by appraisers, a subscript is used to specify which capitalization rate is intended. An equity capitalization rate, therefore, is written as R_E.

Indices

The lowest factors of 2000 are 2×2×2×2×5×5×5. These factors are written as $2^4 \times 5^3$, where 2 and 5 are called bases and the numbers 4 and 5 are called indices. When an index is an integer it is called a power. Thus, 24 is called 'two to the power of four', and has a base of 2 and an index of 4. Similarly, 53 is called 'five to the power of 3' and has a base of 5 and an index of 3. Special names may be used when the indices are 2 and 3, these being called 'squared' and 'cubed', respectively. Thus 7^2 is called 'seven squared' and 9^3 is called 'nine cubed'.

Reciprocal

The reciprocal of a number is when the index is −1 and its value is given by 1, divided by the base. Thus the reciprocal of 2 is 2^{-1} and its value is 1/2 or 0.5. Similarly, the reciprocal of 5 is 5^{-1} which means 1/5 or 0.2.

Square root

The square root of a number is when the index is 1/2, and the square root of 2 is written as $2^{1/2}$ or $\sqrt{2}$. The value of a square root is the value of the base which when multiplied by itself gives the number. Since 3×3 = 9, then $\sqrt{9} = 3$. However, $(-3)\times(-3)=9$, so $\sqrt{9} = -3$. There are always two answers when finding the square root of a number and this is shown by putting both a + and a − sign in front of the answer to a square root problem. Thus $\sqrt{9} = \pm 3$ and $4^{1/2} = \sqrt{4} = \pm 2$, and so on.

Laws of indices

When simplifying calculations involving indices, certain basic rules or laws can be applied, called the laws of indices. These are given below.

(i) When multiplying two or more numbers having the same base, the indices are added. Thus

$$3^2 \times 3^4 = 3^{2+4} = 3^6$$

(ii) When a number is divided by a number having the same base, the indices are subtracted. Thus

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

(iii) When a number which is raised to a power is raised to a further power, the indices are multiplied. Thus

$(3^5)^2 = 3^{5\times 2} = 3^{10}$

(iv) When a number has an index of 0, its value is 1. Thus

$3^0 = 1$

(v) A number raised to a negative power is the reciprocal of that number raised to a positive power. Thus

$3^{-4} = 1/3^4$ Similarly, $1/2^{-3} = 2^3$

(vi) When a number is raised to a fractional power the denominator of the fraction is the root of the number and the numerator is the power. Thus

$$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{2^{3\times 2}} = \sqrt[3]{2^6} = 2^{6/3} = 2^2 = 4$$

and

$$25^{1/2} = \sqrt[2]{25^1} = \sqrt{25^1} = \pm 5$$

(Note that $\sqrt{\ } \cong \sqrt[2]{\ }$

Standard form

A number written with one digit to the left of the decimal point and multiplied by 10 raised to some power is said to be written in standard form. Thus: 5837 is written as 5.837×10^3 in standard form, and 0.0415 is written as 4.15×10^{-2} in standard form. When a number is written in standard form, the first factor is called the mantissa and the second factor is called the exponent. Thus the number 5.8×10^3 has a mantissa of 5.8 and an exponent of 10^3.

i. Numbers having the same exponent can be added or subtracted in standard form by adding or subtracting the mantissae and keeping the exponent the same. Thus:

$2.3 \times 10^4 + 3.7 \times 10^4$

$= (2.3 + 3.7) \times 10^4 = 6.0 \times 10^4$

and

$5.9 \times 10^{-2} - 4.6 \times 10^{-2}$

$= (5.9 - 4.6) \times 10^{-2} = 1.3 \times 10^{-2}$

When the numbers have different exponents, one way of adding or subtracting the numbers is to express one of the numbers in non-standard form, so that both numbers have the same exponent. Thus:

$2.3 \times 10^4 + 3.7 \times 10^3$

$= 2.3 \times 10^4 + 0.37 \times 10^4$

$= (2.3 + 0.37) \times 10^4$

$= 2.67 \times 10^4$

Alternatively,

$2.3 \times 10^4 + 3.7 \times 10^3$

$= 23\ 000 + 3700$

$= 26700$

$= 2.67 \times 10^4$

ii. The laws of indices are used when multiplying or dividing numbers given in standard form. For example,

$(2.5 \times 10^3) \times (5 \times 10^2)$

$= (2.5 \times 5) \times (10^{3+2})$

$= 12.5 \times 10^5$ or 1.25×10^6

Similarly,

$$\frac{6 \times 10^4}{1.5 \times 10^2} = \frac{6}{1.5} \times \left(10^{4-2}\right) = 4 \times 10^2$$

Engineering notation and common prefixes

Engineering notation is similar to scientific notation except that the power of ten is always a multiple of 3.

For example, $0.00035 = 3.5 \times 10^{-4}$ in scientific notation, but

$0.00035 = 0.35 \times 10^{-3}$ or 350×10^{-6} in engineering notation.

Units used in engineering and science may be made larger or smaller by using prefixes that denote multiplication or division by a particular amount. The eight most common multiples, with their meaning, are listed in Table 1, where it is noticed that the prefixes involve powers of ten which are all multiples of 3:

For example,

5MV means $5 \times 1{,}000{,}000 = 5 \times 10^6$

= 5,000,000 volts

3.6 kΩ means $3.6 \times 1000 = 3.6 \times 10^3$

= 3600 ohms

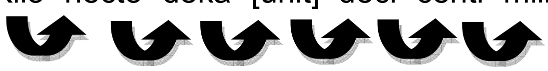
Table 3: Prefix Type

Prefix	Name	Means
T	Tera	Multiply by 1,000,000,000,000
G	Giga	Multiply by 1,000,000,000
M	Mega	Multiply by 1,000,000
k	Kilo	Multiply by 1,000
m	Mili	Divide by 1,000
μ	Micro	Divide by 1,000,000
n	Nano	Divide by 1,000,000,000
p	Pico	Divide by 1,000,000,000,000

Unit conversions within a system

Metric units are very nice to work with, since they are all multiples of ten (or a hundred, or one tenth, etc) of each other. The basic metric units are meters (for length), grams (for mass or weight), and liters (for volume). And the different units convert into one another rather nicely, with one milliliter equalling one cubic centimeter "cc" and one gram being the weight of one cc of water. There are many metric unit prefixes, but the usual ones required in school are these: kilo, hecto, deka, deci, centi, and milli. "Usually" in the middle standing for the "unit", being meters, grams, or liters.

Kilo- hecto- deka- [unit] deci- centi- milli-



x10 x10 x10 x10 x10 x10

1 kilometer = 10 hectometers = 100 dekameters = 1000 meters = 10,000 decimeters = 100,000 centimeters = 1,000,000 millimeters

Alternatively,

1 milliliter = 0.1 centiliters = 0.01 deciliters = 0.001 liters = 0.0001 dekaliters = 0.00001 hectoliters = 0.000001 kiloliters

It is often necessary to make calculations from various conversion tables and charts. Examples include currency exchange rates, imperial to metric unit conversions, train or bus timetables, production schedules and so on.

<u>Time</u>

1 minute (min) = 60 seconds (sec)

1 hour (hr) = 60 minutes (min)

1 day = 24 hours (hr)

1 week (wk) = 7 days

<u>Weight</u>

1 pound (lb) = 16 ounces (oz)

1 ton (T) = 2,000 pounds (lb)

1 ounce ≈ 28.350 grams

1 pound ≈ 453.592 grams

1 milligram = 0.001 grams

1 kilogram = 1,000 grams

1 kilogram ≈ 2.2 pounds

Length

1 foot (ft) = 12 inches (in)

1 yard (yd) = 3 feet (ft)

1 mile (mi) = 5,280 feet (ft)

1 inch = 2.54 centimeters

1 foot = 0.3048 meters

1 meter = 1,000 millimeters

1 meter = 100 centimeters

1 kilometer = 1,000 meters

1 kilometer ≈ 0.62 miles

1 mile =1.61 km

Capacity

1 tablespoon (tbsp) = 3 teaspoons (tsp)

1 cup (c) = 16 tablespoons (tbsp)

1 cup (c) = 8 fluid ounces (fl oz)

1 pint (pt) = 2 cups (c)

1 quart (qt) = 2 pints (pt)

1 gallon (gal) = 4 quarts (qt)

1 quart = 4 cups

1 gallon = 231 cubic inches

1 gallon = 0.003785 m^3

1 liter ≈ 0.264 gallons

1 cubic foot = 1,728 cubic inches

1 cubic yard = 27 cubic feet

1 board foot = 1 inch by 12 inches by 12 inches

1 litre = 1.76 pints

1 gallon = 8 pints

1 m^3 = 35.315 ft^3

Area

1 square foot = 144 square inches

1 square yard = 9 square feet

1 acre = 43,560 square feet

Electricity

1 kilowatt-hour = 1,000 watt-hours

Geometry

Geometry is a part of mathematics in which the properties ofpoints, lines, surfaces and solids are investigated. An angle is the amount of rotation between two straight lines. Angles may be measured in either degrees or radians.

1 revolution=360 degrees, thus 1 degree= 1/360th of one revolution. Also 1 minute = 1/60th of a degree and 1 second = 1/60^{th} of a minute. 1 minute is written as 1 and 1 second is written as 1Thus 1◦ = 60' and 1' = 60"

Example 1: Add 14◦53' and 37◦19'

$$\begin{array}{c} 14^\circ 53' \\ 37^\circ 19' \\ \hline 52^\circ 12' \\ \hline 1^\circ \end{array}$$

$53' + 19' = 72'$. Since $60' = 1^\circ$, $72 = 1^\circ 12'$. Thus the 12 is placed in the minutes column and 1° is carried in the degrees column.

Then $14^\circ + 37^\circ + 1^\circ$ (carried) $= 52^\circ$

Thus $14^\circ 53' + 37^\circ 19' = 52^\circ 12'$

Example 2: Subtract $15^\circ 47'$ from $28^\circ 13'$

$$\begin{array}{l} 27^\circ \\ \cancel{28}^\circ 13' \\ \underline{15^\circ 47'} \\ \underline{12^\circ 26'} \end{array}$$

$13' - 47'$ cannot be done. Hence 1° or 60 is 'borrowed' from the degrees column, which leaves 27° in that column.

Now $(60' + 13') - 47' = 26'$, which is placed in the minutes column.

$27^\circ - 15^\circ = 12^\circ$, which is placed in the degrees column.

Thus $28^\circ 13' - 15^\circ 47' = 12^\circ 26'$

Mesuration

Mesuration is a branch of mathematics concerned with the determination of lengths, areas and volumes.

A polygon is a closed plane figure bounded by straight lines. A polygon, which has:

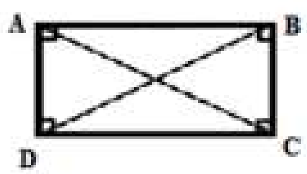
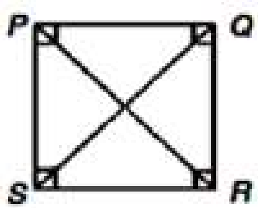
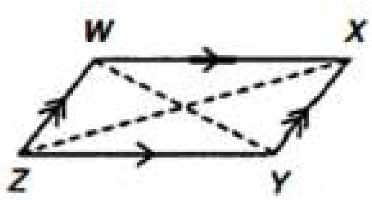
a. 3 sides is called a triangle
b. 4 sides is called a quadrilateral
c. 5 sides is called a pentagon
d. 6 sides is called a hexagon
e. 7 sides is called a heptagon
f. 8 sides is called an octagon

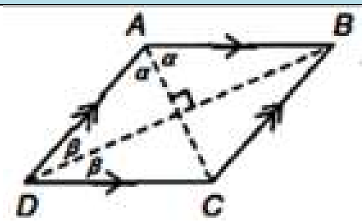
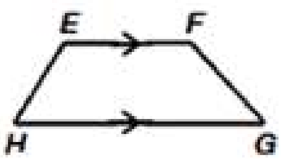
There are five types of quadrilateral, these being:

a. rectangle
b. square
c. parallelogram
d. rhombus
e. trapezium

The properties of these are given below

Table 4: the properties of quadrilateral

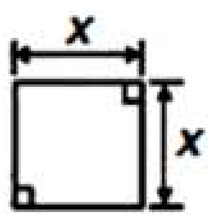
Quadrilateral	Properties
Rectangle 	• all four angles are right angles, • opposite sides are parallel and equal in length, • diagonal *AC* and *BD* are equal in length and bisect one another.
Square 	• all four angles are right angles, • opposite sides are parallel, • all four sides are equal in length, • diagonals PR and QS are equal in length and bisect one another at right angles.
Parallelogram 	• opposite angles are equal, • (opposite sides are parallel and equal in length, • diagonals WY and XZ bisect one another.
Rhombus	• opposite angles are equal,

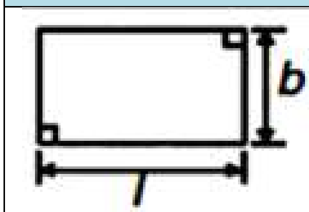
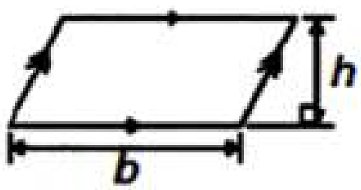
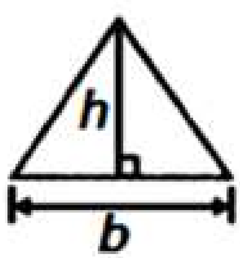
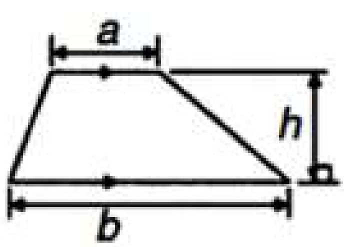
Quadrilateral	Properties
	• opposite angles are bisected by a diagonal, • opposite sides are parallel • all four sides are equal in length, • diagonals AC and BD bisect one another at right angles.
Trapezium 	• only one pair of sides is parallel

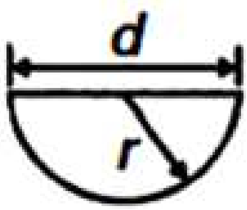
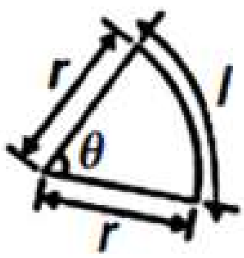
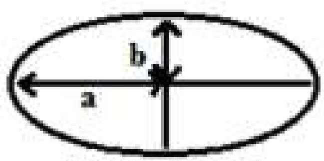
Area

Area is measured in "square" units. The area of a figure is the number of squares required to cover it completely, like tiles on a floor. Table 3 give the summaries of the areas of common plane figures.

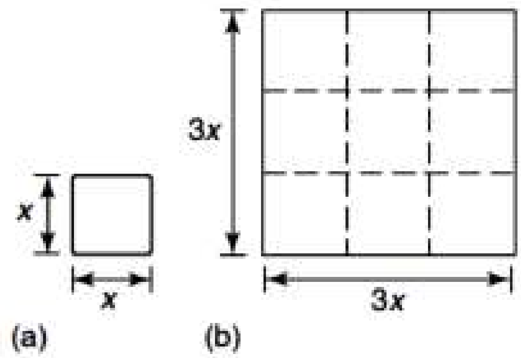
Table 5: Equation of area

Quadrilateral	Equation of Area
Square 	$A = x^2$ Perimeter = 4x
Rectangle	A = l x b perimeter = 2(l + b)

Quadrilateral	Equation of Area
	
Parallelogram 	A = b x h
Triangle 	A = ½ x b x h sum of angles = 180°
Trapezium 	A = ½ (a + b)h
Circle	A = πr^2 or A = ¼ πd^2 Circumstance = πd

Quadrilateral	Equation of Area
	
Semicircle 	A = ½ πr^2 or A = 1/8 πd^2
Sector of a circle 	$A = \frac{\theta^o}{360^o}\left(\pi r^2\right)$
Ellipse 	A = π x a x b Perimeter = π(a + b)

The areas of similar shapes are proportional to the squares of corresponding linear dimensions. For example, Figure below shows two squares, one of which has sides three times as long as the other.



Area of Figure (a) = (x)(x) = x^2

Area of Figure (b) = (3x)(3x) = $9x^2$

Hence Figure (b) has an area $(3)^2$, i.e. 9 times the area of Figure (a).

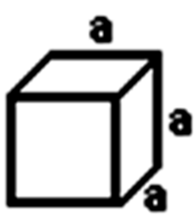
Volumes and Surface Area

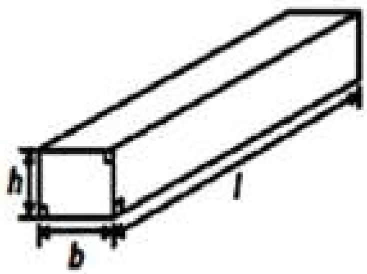
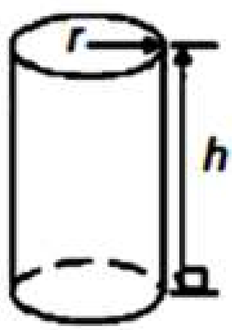
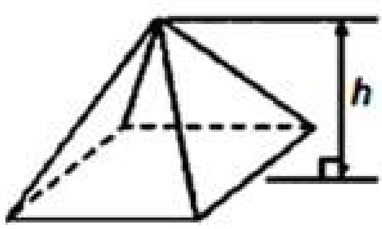
Volume is measured in "cubic" units. The volume of a figure is the number of cubes required to fill it completely, like blocks in a box. Be sure to use the same units for all measurements.

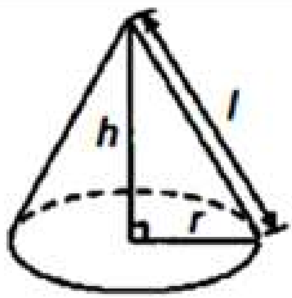
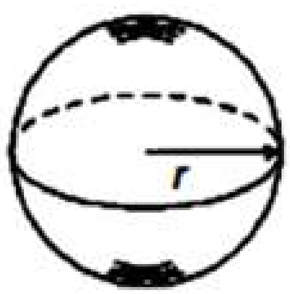
The surface area is the sum of all the areas of all the shapes that cover the surface of the object.

A summary of volumes and surface areas of regular solids is shown in Table 6.

Table 6: summary of volumes and surface areas

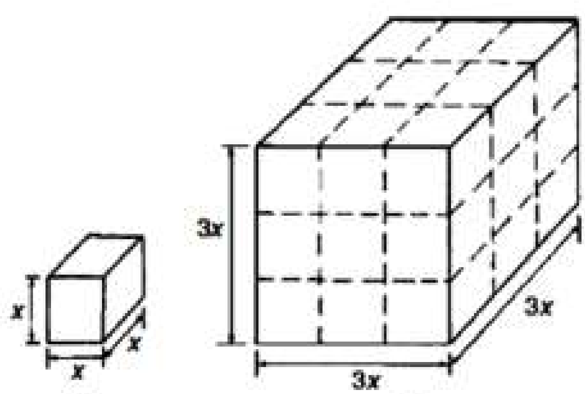
Cube 	$V = a^3$ $A_S = 6a^2$ The surface area of a cube is the area of the six squares that cover it

Rectangular prism (or box) 	$V = A_{base} \times h$ or $V = l \times b \times h$ $A_S = 2(bh + hl + lb)$ The surface area of a rectangular prism is the area of the six rectangles that cover it.
Cylinder 	$V = \pi \times r^2 \times h$ $A_{curved} = 2\pi \times r \times h$ $A_S = A_{curved} + A_{base}$ or $A_S = 2\pi rh + 2\pi r^2$ The surface area is the areas of all the parts needed to cover the can. That's the top, the bottom, and the paper label that wraps around the middle.
Pyramid 	$V = 1/3 \times A \times h$ where A=area of base h=perpendicular height $A_S = 4A_{triangles} + A_{base}$
Cone	$V = 1/3\ \pi \times r^2 \times h$

	$A_{curved} = \pi \times r \times l$ $A_S = A_{curved} + A_{base}$ or $A_S = (\pi \times r \times l) + (\pi \times r^2)$
Sphere 	$V = 4/3\ \pi \times r^3$ $A_S = 4 \times \pi \times r^2$

Volumes of similar shapes

The volumes of similar bodies are proportional to the cubes of corresponding linear dimensions. For example, Figure 7 shows two cubes, one of which has sides three times as long as those of the other.



(a) (b)

Figure 4: Cubes which have similar bodies

Volume of Fig…(a) = (x)(x)(x)=x^3

Volume of Fig…(b) = (3x)(3x)(3x)=$27x^3$

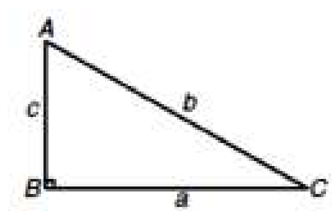
Triangles

<u>Theorem of Pythagoras</u>

With reference to Figure below, the side opposite the right angle (i.e. side b) is called the hypotenuse. The theorem of Pythagoras states:

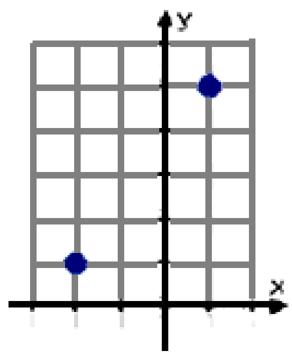
‘In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.’

$$b^2 = a^2 + c^2$$



The Distance Formula

The Distance Formula is a variant of the Pythagorean Theorem that used back in geometry. When given the two points, (x_1, y_1) and (x_2, y_2), the length of the hypotenuse is the distance between the two points.



The distance between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Statistics

Data are obtained largely by two methods:

(a) by counting—for example, the number of stamps sold by a post office in equal periods of time, and

(b) by measurement — for example, the heights of a group of people.

When data are obtained by counting and only whole numbers are possible, the data are called discrete. Measured data can have any value within certain limits and are called continuous. For example

(a) The number of days on which rain falls in a given month must be an integer value and is obtained by counting the number of days. Hence, these data are discrete.

(b) A salesman can travel any number of miles (and parts of a mile) between certain limits and these data are measured. Hence the data are continuous.

(c) The time that a battery lasts is measured and can have any value between certain limits. Hence these data are continuous.

(d) The amount of money spent on food can only be expressed correct to the nearest pence, the amount being counted. Hence, these data are discrete.

A set is a group of data and an individual value within the set is called a member of the set. A set containing all the members is called a population. Some member selected at random from a population are called a sample.

The number of times that the value of a member occurs in a set is called the frequency of that member. Thus, in the set:
{2, 3, 4, 5, 4, 2, 4, 7, 9}, member 4 has a frequency of three, member 2 has a frequency of 2 and the other members have a frequency of one.

The relative frequency with which any member of a set occurs is given by the ratio:

$$\frac{frequency\ \ of\ \ member}{total\ \ frequency\ \ of\ \ all\ \ member}$$

For the set: {2, 3, 5, 4, 7, 5, 6, 2, 8}, the relative frequency of member 5 is 2/9

Often, relative frequency is expressed as a percentage and the percentage relative frequency is:

(relative frequency × 100)%

<u>Presentation of ungrouped data</u>

Ungrouped data can be presented diagrammatically in several ways and these include:

(a) pictograms, in which pictorial symbols are used to represent quantities

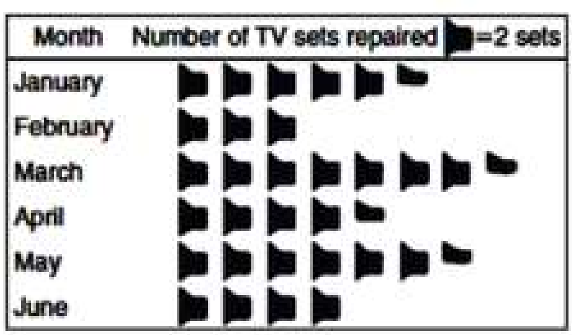


Figure 5: Pictograms

(b) horizontal bar charts, having data represented by equally spaced horizontal rectangles

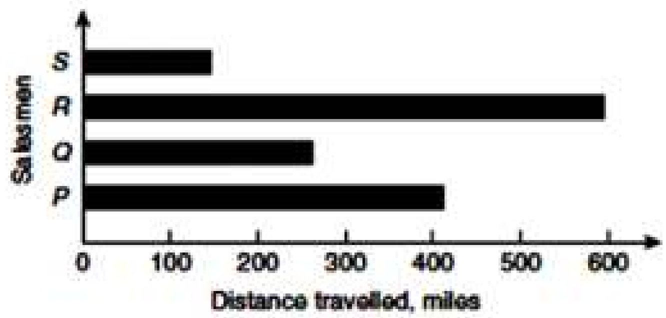


Figure 6: Horizontal bar charts

(c) vertical bar charts, in which data are represented by equally spaced vertical rectangles

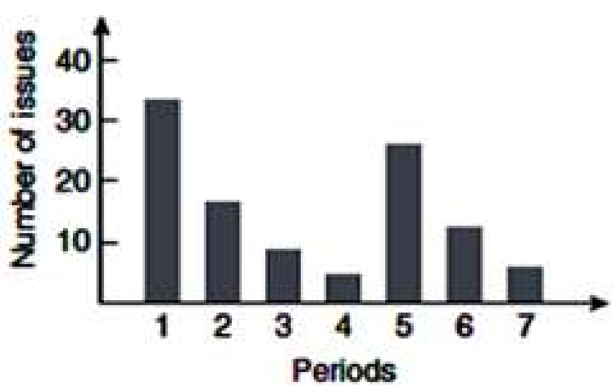


Figure 7: Vertical bar charts

Trends in ungrouped data over equal periods of time can be presented diagrammatically by a percentage component bar chart. In such a chart, equally spaced rectangles of any width, but whose height corresponds to 100%, are constructed. The rectangles are then subdivided into values corresponding to the percentage relative frequencies of the members. A pie diagram is used to show diagrammatically the parts making up the whole.

In a pie diagram, the area of a circle represents the whole, and the areas of the sectors of the circle are made proportional to the parts which make up the whole.

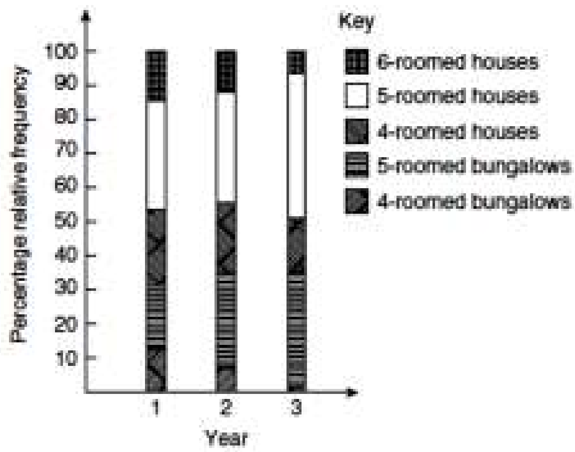


Figure 8: Percentage component bar chart

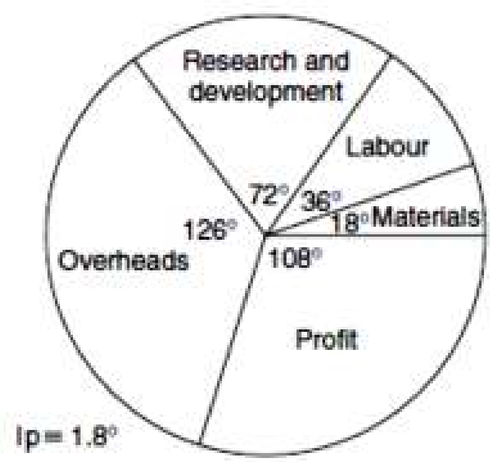


Figure 9: Pie diagram

In pie diagram if you want to calculate the amount of part, it can used by

If it used degree

$$amount\ \ A = \frac{A^O}{360^O} \times total\ \ amount$$

If it used percentage

$$amount\ \ A = \frac{A\%}{100\%} \times total\ \ amount$$

Measures in statistic

A single value, which is representative of a set of values, may be used to give an indication of the general size of the members in a set, the word 'average' often being used to indicate the single value. The statistical term used for 'average' is the arithmetic mean or just the mean. Other measures of central tendency may be used and these include the median and the modal values.

Mean

The arithmetic mean value is found by adding together the values of the members of a set and dividing by the number of members in the set. In general, the mean of the set: $\{x_1, x_2, x_3, \ldots, x_n\}$ is

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$$ written as

$$\bar{x} = \frac{\sum x}{n}$$

where

Σ = 'sigma' means 'the sum of'

$\bar{x}$ = used to signify a mean value

Median

The median value often gives a better indication of the general size of a set containing extreme values.

The median value is obtained by:

(a) ranking the set in ascending order of magnitude,

(b) selecting the value of the middle member for sets containing an odd number of members, or finding the value of the mean of the two middle members for sets containing an even number of members.

For example, the set: {7, 5, 74, 10} is ranked as {5, 7, 10, 74}, and since it contains an even number of members (four in this case), the mean of 7 and 10 is taken, giving a median value of 8.5.

Mode

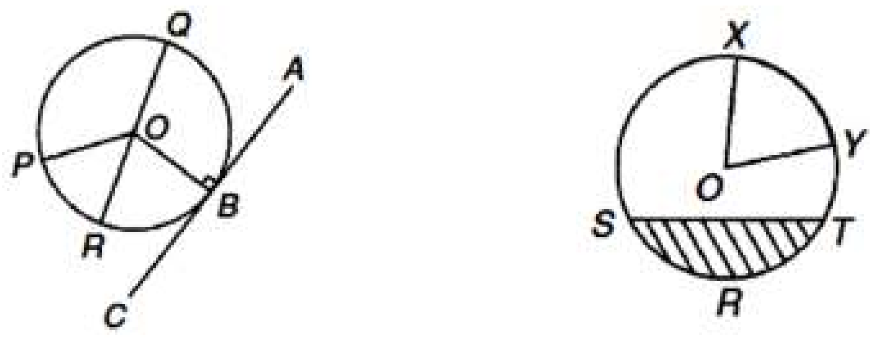
The modal value, or mode, is the most commonly occurring value in a set. If two values occur with the same frequency, the set is ‘bi-modal’.

For example {5, 6, 8, 2, 5, 4, 6, 5, 3} has a modal value of 5, since the member having a value of 5 occurs three times.

Circle

A circle is a plain figure enclosed by a curved line, every point on which is equidistant from a point within, called the centre.

<u>Properties of circles</u>



(a) (b)

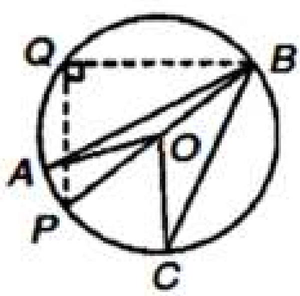
Figure 10: Circle Properties

1. The distance from the centre to the curve is called the radius, *r*, (OQ or OR)
2. The boundary of a circle is called the circumference, c.

   $c = 2 \times \pi \times \text{radius} = 2\pi r$ or

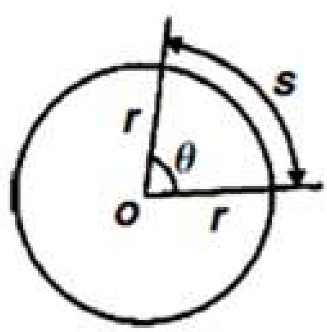
   $c = \pi \times \text{diameter} = \pi d$
3. Any straight line passing through the centre and touching the circumference at each end is called the diameter, *d*(QR). Thus d = 2r
4. A semicircle is one half of the whole circle.
5. A quadrant is one quarter of a whole circle.
6. A tangent to a circle is a straight line which meets the circle in one point only and does not cut the circle when produced. *AC* in Figure10 (a) is a tangent to the circle since it touches the curve at point *B* only. If radius *OB* is drawn, then angle *ABO* is a right angle.
7. A sector of a circle is the part of a circle between radii (for example, the portion *OXY* of Figure 10(b) is a sector). If a sector is less than a semicircle it is called a minor sector, if greater than a semicircle it is called a major sector.
8. A chord of a circle is any straight line which divides the circle into two parts and is terminated at each end by the circumference. *ST*, in Figure 10 (b) is a chord.
9. A segment is the name given to the parts into which a circle is divided by a chord. If the segment is less than a semicircle it is called a minor segment (see shaded area in Figure 10). If the segment is greater than a semicircle it is called a major segment (see the unshaded area in Figure 10).

10. An arc is a portion of the circumference of a circle. The distance *SRT* in Figure 10 is called a minor arc and the distance *SXYT* is called a major arc.
11. The angle at the centre of a circle, subtended by an arc, is double the angle at the circumference subtended by the same arc. With reference to Figure 10, Angle *AOC* =2×angle *ABC*.
12. The angle in a semicircle is a right angle (see angle *BQP* in Figure below).



<u>Arc length and area of a sector</u>

One radian is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius.



θ radians = s/r

Where θ is in radian and s is arc length.

2π radians = 360^o thus π radians = 180^o
Since π rad=180°, then $\pi/2=90^\circ$, $\pi/3=60^\circ$, $\pi/4=45^\circ$, and so on.

When θ is in degrees:

Area of a sector = $\frac{\theta}{360}\left(\pi r^2\right)$

When θ is in radians:

Area of a sector = $\frac{\theta}{2\pi}\left(\pi r^2\right) = \frac{1}{2} r^2 \theta$