International Association Of Certified Practicing Engineers	www.iacpe.com Knowledge, Certification, Networking	Rev 01 - Oct 2014
IACPE No 19, Jalan Bilal Mahmood 80100 Johor Bahru Malaysia	**SOLID AND FLUID MECHANICS** **CPE LEVEL I** **TRAINING MODULE**	

The International Association of Certified Practicing Engineers is providing the introduction to the Training Module for your review.

We believe you should consider joining our Association and becoming a Certified Practicing Engineer. This would be a great option for engineering improvement, certification and networking.

This would help your career by

1. Providing a standard of professional competence in the practicing engineering and management field
2. Identify and recognize those individuals who, by studying and passing an examination, meets the standards of the organization
3. Encourage practicing engineers and management professionals to participate in a continuing program of personal and professional development

www.IACPE.com

TABLE OF CONTENTS

LIST OF TABLE

LIST OF FIGURE

INTRODUCTION

Scope

Solid mechanics is the study of the deformation and motion of solid materials under the action of forces, temperature changes, phase changes, and other external or internal agents. It is one of the fundamental applied engineering sciences in the sense that it is used to describe, explain and predict many of the physical phenomena.

Solid mechanics is typically useful in designing and evaluating tools, machines, and structures, ranging from wrenches to cars to spacecraft. The required educational background for these includes courses in statics, dynamics, and related subjects. For example, dynamics of rigid bodies is needed in generalizing the spectrum of service loads on a car, which is essential in defining the vehicle's deformations and long-term durability.

Fluid mechanics deals with the study of all fluids under static and dynamic situations. Fluid mechanics is a branch of continuous mechanics which deals with a relationship between forces, motions, and statistical conditions in a continuous material. This study area deals with many and diversified problems such as fluid statics, flow in enclose bodies, or flow round bodies (solid or otherwise), flow stability, etc.

Both solid mechanics and fluid mechanics play very important roles in design. Because a fluid cannot resist deformation force, it moves, or flows under the action of the force. Its shape will change continuously as long as the force is applied. Whereas, a solid can resist a deformation force while at rest. While a force may cause some displacement, the solid does not move indefinitely.

Solid mechanics which is based on Newton laws, either in rest or motion. Solid mechanics consist of several fundamentals such as vectors, moments, couple, moment inertia, motion, vibration, and rigid bodies in statics and dynamics. Fluid mechanics consist of fluid properties and hydrostatic forces along with their application in nature.

INTRODUCTION

General Consideration

A. Aspects of Solid Mechanics

The theory of solid mechanics starts with the particle, and then a rigid body. Rigid body mechanics is usually subdivided into statics and dynamics.

Statics

Statics are the study of materials at rest. The actions of all external forces acting on such materials are exactly counterbalanced and there is a zero net force effect on the material: such materials are said to be in a state of static equilibrium. Equilibrium is said to be stable when the body with the forces acting upon it returns to its original position after being displaced a very small amount from that position; and neutral when the forces retain their equilibrium when the body is in its new position.

If a body is supported by other bodies while subject to the action of forces, deformations and forces will be produced at the points of support or contact and these internal forces will be distributed throughout the body until equilibrium exists. They are equal in magnitude and opposite in direction to the forces with which the supports act on the body, known as supporting forces. The supporting forces are external forces applied to the body[6].

A material body can be considered to consist of a very large number of particles. A rigid body is one which does not deform, in other words, the distance between the individual particles making up the rigid body remains unchanged under the action of external forces. An example of the statics of a rigid body is a bridge supporting the weight a car.

Dynamics

There are two major categories in dynamics, kinematics and kinetics. Kinematics involves the time and geometry-dependent motion of a particle, rigid body, deformable body, or a fluid without considering the forces that cause the motion. It relates position, velocity, acceleration, and time. Kinetics combines the concepts of kinematics and the forces that cause the motion.

Dynamics of a rigid body follows Newton's second law. Newton's second law of motion states that the body will accelerate in the direction of and proportional to the magnitude of the resultant R. In rectilinear motion, the acceleration and the direction of the unbalanced force must be in the direction of motion. Forces must be in balance and the acceleration equal to zero in any direction other than the direction of motion. An example of the dynamics of rigid body is an accelerating and decelerating elevator.

General Laws

The fundamental concepts and principles of mechanics follow the relation between the motion and the force that is defined by Newton's Law. Newton's law states that:

1. A body remains at rest or continues in a straight line at a constant velocity unless acted upon by an external force.
2. A force applied to a body accelerates the body by an amount which is proportional to the force.
3. Every action is opposed by an equal and opposite reaction.

B. Aspect of Fluid Mechanics

Fluid mechanics is a study of the relationships between the effects of forces, energy and momentum occurring in and around a fluid system. Fluids are substances capable of following and taking the shape of containers. Fluids can be classified as liquids or gases; liquids are incompressible, occupy definite volumes, and have free surfaces; whereas, gases are compressible and expand until they occupy all portions of the container. Fluids cannot sustain shear or tangential forces when in equilibrium.

Substances may be classified by their response when at rest to the imposition of a shear force. Liquid that undergoes a shear stress between a short distance of two plates can be shown in Figure 1.

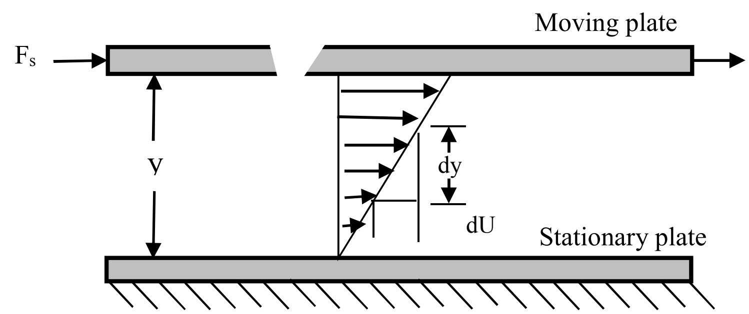


Figure 1 : schematics to describe the shear stress in fluid mechanics

The deformation characteristics of various substances are divided to five characteristics as illustrated in figure 2. An ideal or elastic solid will resist the shear force, and its rate of deformation will be zero regardless of loading and hence is coincident with the ordinate. A plastic will resist the shear until its yield stress is attained, and the application of additional loading will cause it to deform continuously, or flow. If the deformation rate is directly proportional to the flow, it is called an ideal plastic.

A Newtonian fluid is a real fluid which has internal friction so that their rate of deformation is proportional to the applied shear stress. If it is not directly proportional, it is called a non-Newtonian fluid.

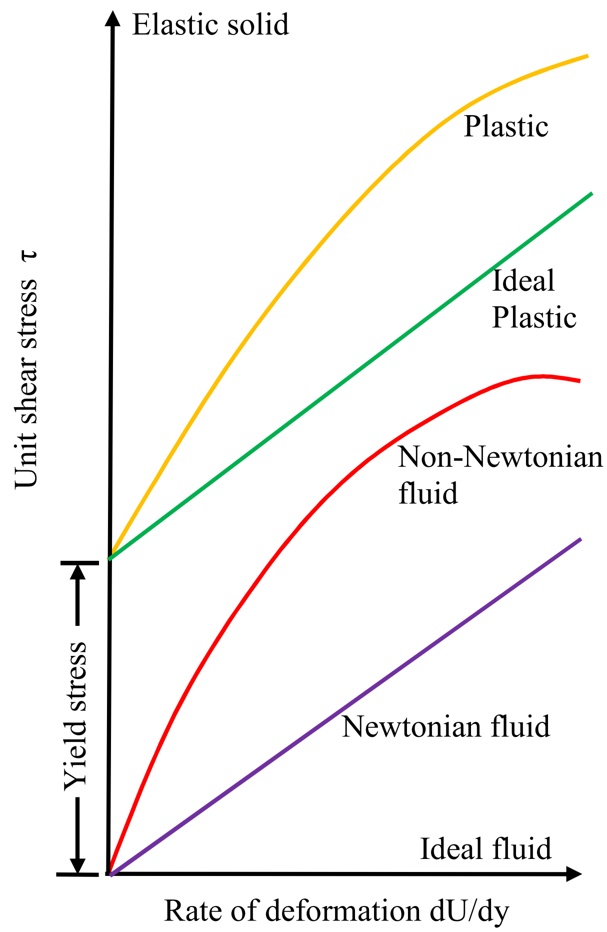


Figure 2 : deformation characteristics of substances

Fluid Statics

In a static fluid, an important property is the pressure in the fluid. Pressure is defined as force exerted by a mass under the influence of gravity and a mass of fluid on a supporting area, or force per area. The fluid pressure acts normal to any plane and is transmitted with equal intensity in all directions. In fluid mechanics and in thermodynamic equations, the units are lbf/ft^2, but engineering practice is to use units of lbf/in^2.

Most fluid-mechanics equations and all thermodynamic equations require the use of absolute pressure, and unless otherwise designated, a pressure should be understood to be absolute pressure. Common practice is to denote absolute pressure as lbf/ft^2 abs, or psfa, lbf/in^2 abs or psia; and in a like manner for gauge pressure lbf/ft^2 g, lbf/in^2 g, and psig. The relationship between absolute pressure, gauge pressure, and vacuum is shown in Figure 3 [6].

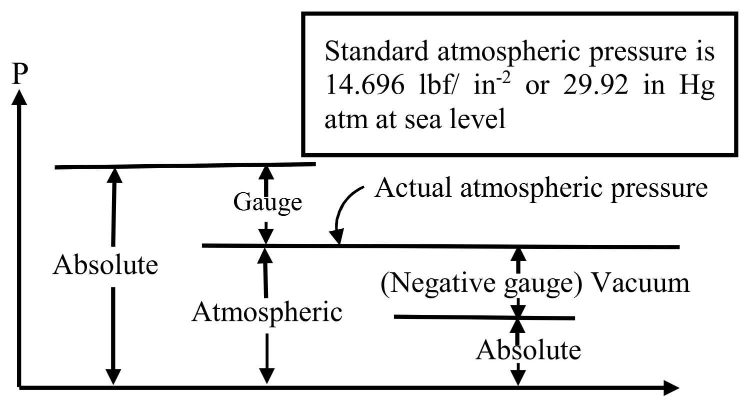


Figure 3 : pressure relation

Fluid Dynamics

Fluid dynamics is a sub-discipline of fluid mechanics that deals with fluid flow – natural science of fluid in motion. The elements of a flowing fluid can move at different velocities and can be subjected to different accelerations. The following principles apply in fluid flow:

a. The principle of conservation of mass, from which the equation of continuity is developed.
b. The principle of kinetic energy, from which some flow equations are derived.
c. The principle of momentum, from which equations regarding the dynamic forces exerted by flowing fluids can be established.

Types of Fluid

Fluid flow can be characterized as steady or unsteady, uniform or non-uniform. There typically can classify any flow as follow.

1. Steady uniform flow

   Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

2. Steady non-uniform flow

   Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as moving along the length of the pipe toward the exit.

3. Unsteady uniform flow

   At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

4. Unsteady non-uniform flow

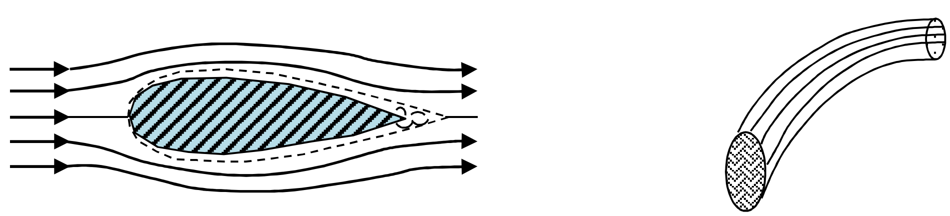
   Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

In additional, there are also considered about number of dimensional required to describe the velocity profile.

1. Flow is one dimensional, if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time only vary in the direction of flow and not across the cross-section.
2. Flow is two-dimensional, if it can be assumed that the flow parameters vary in the direction of flow and in one direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. The example is area.
3. All flows take place between boundaries that are three-dimensional. Typically, it takes three directions of flow such as x, y, z as a volume of fluid in a circular pipe.

Streamlines and Stream-tube

A streamline is a line which gives the direction of the velocity of a fluid particle at each point in the flow stream. When streamlines are connected by a closed curve in steady flow, they will form a boundary through which the fluid particles cannot pass. The space between the streamlines becomes a stream tube. They can be illustrated in figure 4.



(a) streamlines around a wing (b) stream-tube

Figure 4 : streamlines and stream-tube

The Laws of Thermodynamics

The First Law of Thermodynamics

This is a law of energy conservation. It states that energy is always conserved, it cannot be created or destroyed. In essence, energy can be converted from one form into another[9]. The increase in internal energy of a closed system can be defined as follows.

$$Q - W = \Delta U \qquad \text{Eq (1)}$$

Where,

- Q = heat transfer, Btu (kJ)
- W = work transfer, Btu (kJ)
- ΔU = increase in internal energy, Btu (kJ)

For a thermodynamic cycle of a closed system, which returns to its original state, the heat Q_{in} supplied to a closed system in one stage of the cycle, minus that Q_{out} removed from it in another stage of the cycle, equals the net work done by the system. Work done by a system is considered positive; $W > 0$. Work done on a system is considered negative; $W < 0$.

The Second Law of Thermodynamics

It might be thought that the first law of thermodynamics permits all the heat transfer to a cycle to be returned as work transfer, but unfortunately the second law places restraints on the achievement of this desirable situation. It states that in all energy exchanges, if no energy enters or leaves the system, the potential energy of the state will always be less than that of the initial state. This is also commonly referred to as entropy. Entropy is a measure of disorder[13].

DEFINITION

Angular displacement - a particle moves on a circular path its angle of rotation (or its angular displacement) θ, varies with time.

Boundary layer - thin layer of fluid adjacent to a surface where viscous effects are important; viscous effects are negligible outside the boundary layer.

Buoyancy - is based on Archimedes' principle, which states that the buoyant force exerted on a submerged body is equal to the weight of the displaced fluid.

Center of gravity - the point through which the whole weight of a body may be assumed to act.

Couple - pair of two equal and opposite forces acting on a body in a such a way that the lines of action of the two forces are not in the same straight line.

Drag coefficient - force in the flow direction exerted on an object by the fluid flowing around it, divided by dynamic pressure and area.

Energy - the capacity of a body to do work by reason of its motion or configuration.

Entropy - a measure of the disorder of any system, or of the unavailability of its heat energy for work.

Force - the action of one body on another which will cause acceleration of the second body unless acted on by an equal and opposite action counteracting the effect of the first body.

Friction - the resistance that is encountered when two solid surfaces slide or tend to slide over each other.

Impulse - the product of the force and the time that force acts

Inertia - property of matter which causes a resistance to any change in the motion of a body.

Isentropic - one condition for which there is no heat transfer in reversible between the system and surroundings, therefore this process is also adiabatic.

Lift coefficient - force perpendicular to the flow direction exerted on an object by the fluid flowing around it, divided by dynamic pressure and area.

Linear momentum - the product of mass and the linear velocity of a particle and is a vector.

Metacenter - the point at which the line of action of the buoyancy force cuts the vertical center line of the floating body in the displaced position.

Moment of the force - the turning effect of a force on a body.

Polar moment of inertia - the sum of the moments of inertia about any two axes at right angles to each other in the plane of the area and intersecting at the pole.

Potential energy - the energy possessed by an element of fluid due to its elevation above a reference datum.

Rigid body - one in which the particles are rigidly connected that does not deform, or change shape.

Resonance - characteristic through increasing amplitude to infinity. The resonance phenomenon appears when the frequency of perturbation or forced angular frequency, p is equal to the natural angular frequency ω.

Steady uniform flow - conditions do not change with position in the stream or with time.

Streamline - a line which gives the direction of the velocity of a fluid particle at each point in the flow stream.

Separation - phenomenon that occurs when fluid layers adjacent to a solid surface are brought to rest and boundary layers depart from the surface contour, forming a low pressure wake region. Separation can occur only in an adverse pressure gradient.

Transition - change from laminar to turbulent flow within the boundary layer.

Vector - a directed line segment that has both magnitude and direction.

Viscosity - a measure of the resistance fluids to flow and may be considered as internal friction.

NOMENCLATURE

A	sectional area, ft^2 (m^2)
a	total angular acceleration, rad/s^2
c	actual damping coefficient, N.s/m
c_{cr}	critical damping coefficient, N.s/m
C_D	drag coefficient, dimensionless
Cv	constant volume heat capacity, Btu/lbmol.ºF (J/mol.K)
d	distance, ft (m)
dv	velocity differential, ft/s (m/s)
dy	distance differential, ft (m)
f	frequency, Heartz (rad/s)
F	force, lbf (N)
F_B	buoyant force, lbf (N)
F_D	drag force, lbf (N)
F_H	normal force on the vertical projection, lbf (N)
F_0	amplitude of the forced vibration, dimensionless
F_V	weight of fluid above the curve, lbf (N)
g	acceleration gravitional, 32.2 ft/s^2 (9.81 m/s^2)
h	height above the ground, ft (m)
H	head, ft (m)
H_1	enthalpy at point 1, btu/lb (J/kg)
H_2	enthalpy at point 2, btu/lb (J/kg)
h_A	head added, ft (m)
h_L	head loss, ft (m)
h_E	head extracted, ft (m)
H_O	angular momentum about O, ($kg.m^2/s$)
I	moment of inertia, $lbm.ft^2$ ($kg.m^2$)
k	radii of gyration, ft (m)
k	spring stiffness, N/m
KE	kinetic energy, J (N.m)
m	mass, lb (kg)
M	moment, lbf.ft (Nm)
MF_i	mole fraction of component *i*

MW_i molecular weight of component *i*, lb/ft^3 (kg/m^3)
P power, HP (watt)
PE potential energy, J (N.m)
Q energy added, Btu/lb (J/kg)
r radius of circular path, ft (m)
R gas law constant, 10.731 $ft^3.lb_f/in^2.lbmol\,^oR$ ($8314.34\ kg.m^2/s^2.kgmol.K$)
s space or displacement, ft (m)
S entropy, btu/lbm.^oF (kJ/kg.K)
T period, sec/cycle
T temperature, ^oR (K)
U internal energy, btu (J)
v angular velocity, rad/s
v velocity, ft/s (m/s)
V volume, ft^3 (m^3)
W weight, lbf (N)
W work, J (N.m)
Ws net mechanical work, Btu/lb (J/kg)
z elevation or depth, ft (m)

Greek letters

$\dot{m}$ mass flow rate, lbm/s (kg/s)
α angle of inclination, degree
α angular acceleration, rad/sec^2
α kinetic energy velocity correction factor
a_n normal acceleration, rad/s^2
a_t tangential acceleration, rad/s^2
β momentum velocity correction factor
ε absolute roughness, in (mm)
$\sum F$ total of force, lbf (N)
$\sum F$ frictional losses, Btu/lb (J/kg)
δ polytropic index
θ angular displacement, rad
ω angular velocity, rad/s
ω_d damped natural frequency, Hz (rad/s)
ζ damping factor, ratio c/c_{cr}

ρ radius of curvature of the path, ft (m)
τ shear force, lb/ft.s^2 (N/m^2)
μ viscosity, lbf.s/ft^2 or poise (N.s/m^2)
μ_m gas mixture viscosity, micropoise (N.s/m^2)
ν kinematic viscosity, ft^2/s or stoke (m^2/s)
γ specific weight, lbf/ft^3 (N/m^3)
γ specific heat ratio, Cp/Cv

THEORY

A. Solid Mechanics

I. Fundamental of Solid Mechanic Statics

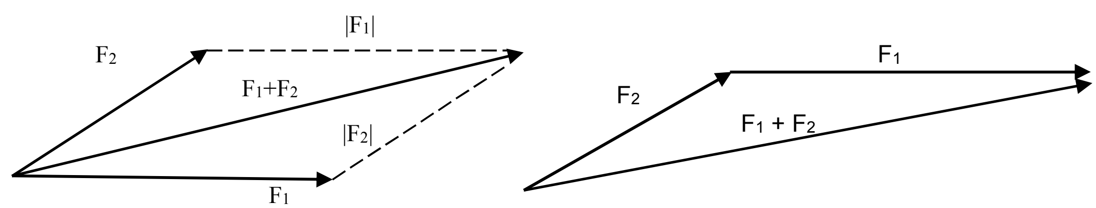
Vectors

A fundamental that should be mastered in statics of a rigid body is the vector. Two kinds of quantities are used in engineering mechanics. A scalar quantity has only magnitude (mass, time, temperature, etc.). A vector quantity has magnitude and direction (force, velocity, etc.). Vectors are represented here by arrows and are used in analysis according to universally applicable rules that facilitate calculations in a variety of problems[10].

Characteristics of Vector

a. Vector Addition

Any number of concurrent vectors may be summed, mathematically or graphically, and in any order. The vectors F_1 and F_2 add according to the parallelogram law: $F_1 + F_2$ is equal to the diagonal of a parallelogram formed by the graphical representation of the vectors as shown in figure 5(a). The vectors also can be added by moving them successively to parallel positions so that the head of one vector connects to the tail of the next vector as shown in figure 5(b).



(a) (b)

Figure 5: vectors addition

b. Resolution of Vectors and Components

A resultant force may be resolved into two forces at right angles to another. The resultant shown is at angle θ to *x* axis as follow.

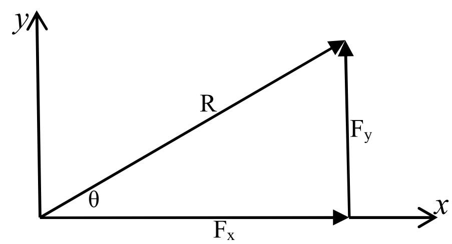


Figure 6: Triangle of forces

There are components and resultant which can be defined as follow.

$$F_x = R \cos \theta \qquad \text{Eq (2)}$$
$$F_Y = R \sin \theta \qquad \text{Eq (3)}$$

Therefore, the angle and resultant can be obtained by the components as below:

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad \text{Eq (4)}$$

$$R = \sqrt{F_x^2 + F_y^2} \qquad \text{Eq (5)}$$

c. Scalar Product of Two Vectors

The scalar (dot) product of two concurrent vectors A and B is defined by

$$a \cdot b = b \cdot a = |a||b|\cos\theta \qquad \text{Eq (6)}$$

where A and B are the magnitudes of the vectors, they are defined as follow.

$$A \cdot B = B \cdot A = A_xB_x + A_yB_y + A_zB_z \qquad \text{Eq (7)}$$

$$\theta = \arccos\frac{A_xB_x + A_yB_y + A_zB_z}{AB} \qquad \text{Eq (8)}$$

For the scalar triple product, this scalar product is used in calculating moments. A, B, C can be expressed in the following determinant form:

$$A\cdot(B\times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = A_x\left(B_yC_z - B_zC_y\right) + A_y\left(B_zC_x - B_xC_z\right) + A_z\left(B_xC_y - B_yC_x\right) \qquad \text{Eq (9)}$$

d. Vector Product of Two Vectors

A powerful method of vector mechanics is available for solving complex problems, such as the moment of a force in three dimensions. The vector (cross) product of a vector A and a vector B is defined as A x B that should be perpendicular to the plane of vectors A and B. There are several rules :

The sense of the unit vector n that appears in the definition of A x B depends on the order of the factors A and B in such a way that

$$A \times B = -(B \times A) \qquad \text{Eq (10)}$$

The magnitude of a x b is given by

$$|A\times B| = |A||B|\sin\theta \qquad \text{Eq (11)}$$

A set of mutually perpendicular unit cordinate vectors i, j, k is called right-handed when i x j = k and left-handed when i x j = -k.

The vector product is calculated using a determinant form as follows.

$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = A_y B_z i + A_z B_x j + A_x B_y k - A_y B_x k - A_x B_z j - A_z B_y i \qquad \text{Eq (12)}$$

For three vectors, the vector triple product of three vectors A, B, C is the vector A x(B x C) as defined by

$$A \times (B \times C) = A \cdot CB - A \cdot BC \qquad \text{Eq (13)}$$

Statics of Rigid Bodies

All solid materials deform when forces are applied to them, but often it is reasonable to model components and structures as rigid bodies, at least in the early part of the analysis. The forces on a rigid body are generally not concurrent at the center of mass of the body, which cannot be modeled as a particle if the force system tends to cause a rotation of the body. Resultant of forces acting on a body can be considered by number of forces, which can be described as follow.

a. Resultant of Forces Acting on a Body at the Same Point

The resultant R of two forces F_1 and F_2 applied to a rigid body at the same point is represented in magnitude and direction by the diagonal of the parallelogram formed by F_1 and F_2. It can be illustrated in figure 7.

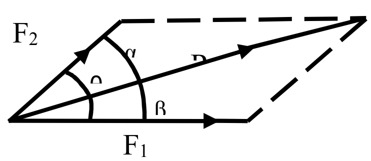


Figure 7: resultant of two forces

The resultant and degrees between the result are defined by

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \qquad \text{Eq (14)}$$

And,

$$\sin \alpha = (F_1 \sin \theta) / R \qquad \text{Eq (15)}$$

$$\sin \beta = (F_2 \sin \theta) / R \qquad \text{Eq (16)}$$

b. Lami's Theorem

If three coplanar forces acting on a point in a body keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces. There are three forces P, Q and R acting at a point O which between each of two forces has angle as shown as follow[1].

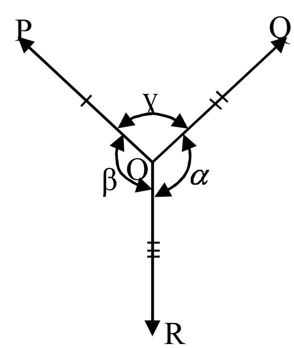


Figure 8 : Lami's theorem

These forces are in equilibrium then according to Lami's theorem that is given as follow.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} \qquad \text{Eq (17)}$$

c. Resultant of Any Number of Forces Applied to a Rigid Body at the Same Point

The three-dimensional components and associated quantities of a vector R as shown in figure 9. The unit vector *n* is collinear with R. A unit vector is a vector with the magnitude equal to 1 of sum of unit cordinate vectors and a defined direction[10].

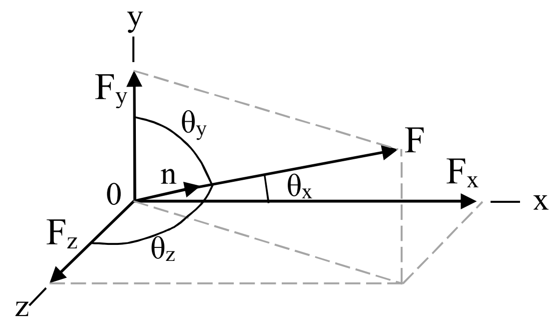


Figure 9 : Three-dimensional components of a vector R

The vector R is written in terms of its scalar components and the unit coordinate vectors as follow.

$$R = F_x i + F_y j + F_z k = Rn \quad \text{Eq (18)}$$

Where, each of components is defined as below.

$$F_x = R \cos \theta_x \quad \text{Eq (19)}$$

$$F_y = R \cos \theta_y \quad \text{Eq (20)}$$

$$F_z = R \cos \theta_z \quad \text{Eq (21)}$$

And, resultant can be obtained by

$$R = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad \text{Eq (22)}$$

However, the method is to find unit vector, *n* on the line of points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) can be obtained by

$$n = \frac{vector\ A\ to\ B}{distance\ A\ to\ B} = \frac{d_x i + d_y j + d_z k}{\sqrt{d_x^2 + d_y^2 + d_z^2}} \qquad \text{Eq (23)}$$

Where,

dx = $x_2 - x_1$
dy = $y_2 - y_1$
dz = $z_2 - z_1$

d. Moment of The Force

Moment of the force, or torque is the turning effect of a force on a body. Moment of force can be considered with respect to a point and straight line that is described as follow.

i. The Moment of a Force with Respect to a Point

The tendency of a force to make a rigid body rotate is measured by the moment of that force about an axis. The moment of a force F about an axis through a point O is defined as the product of the magnitude of F times the perpendicular distance d from the line of action of F and the axis O. It is shown in figure 10.
.

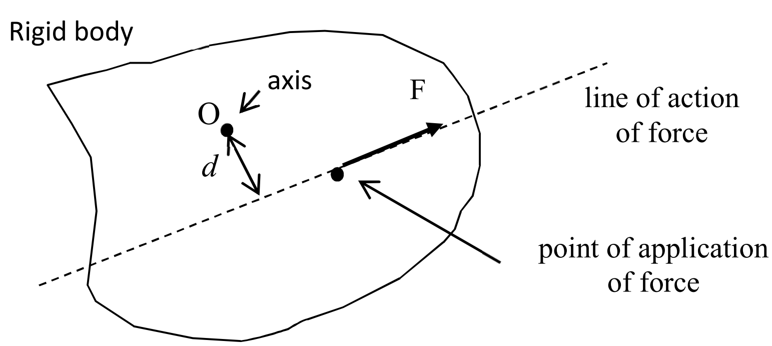


Figure 10 : moment of the force

The equation of this case is defined as a scalar quantity as follow.

$$Mo = F.d \quad \text{Eq (24)}$$

Where,

Mo = moment about O, lbf.ft (Nm)
F = force, lbf (N)
d = distance, ft (m)

Clockwise moment is reckoned positive and counterclockwise moments negative[7]. When the line of action of a force passes through the axis, the moment is zero, Mo = 0 and the system is in equilibrium. Two forces of equal magnitude and acting along the same line of action have not only the same components F_X, F_Y, but have equal moments about any axis. They are called equivalent forces since they have the same effect on a rigid body.

Moment of force also may be considered by vector quantity. The moment of a bound vector, v about a point A may be illustrated in figure 11[8].

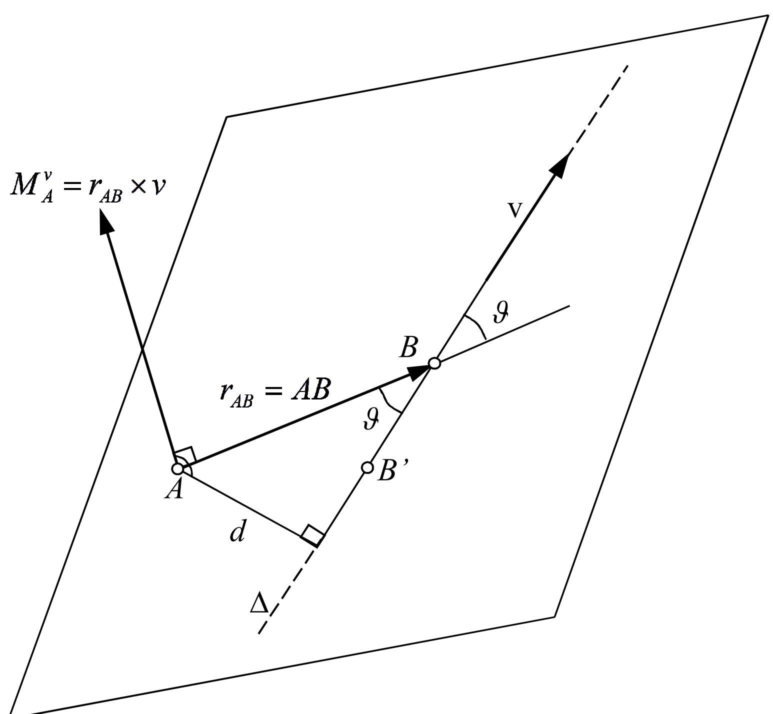


Figure 11 : moment of a bound vector about a point

The vector definition of the moment of this case can be given by

$$M_A^v = AB \times v = r_{AB} \times v \qquad \text{Eq (25)}$$

Where,

r_{AB} = position vector from point A to any point on the line of action of B.

The magnitude of the moment, M_A^v is defined by

$$\left|M_A^v\right| = M_A^v = \left|r_{AB}\right|\left|v\right|\sin\theta \qquad \text{Eq (26)}$$

$$\left|M_A^v\right| = M_A^v = d\left|v\right| \qquad \text{Eq (27)}$$

ii. The Moment of a Force with Respect to a Line

If the force is resolved into components parallel and perpendicular to the given line, the moment of the force with respect to the line is the product of the magnitude of the perpendicular component and the distance from its line of action to the given line. It is common that a body rotates about an axis. In that case the moment of a force about the line is usefully expressed as

$$M_l = n \cdot M_o = n \cdot (r \times F) = \begin{vmatrix} n_x & n_y & n_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \qquad \text{Eq (28)}$$

Where,

n = a unit vector along the line

r = position vector from point *O* on line to a point on the line of action of F

M_l = projection of M_O on line

e. Couple

A moment is called a couple when two equal in magnitude and opposite forces have parallel lines of action a distance a apart. This is independent of *d* and the resultant force is zero. The only motion that a couple can impart is a rotation. In addition, the couple has no tendency to translate a rigid body. The moment about any point 0 at distance *d* from one of the lines of action is expressed as Eq (29)[7].

$$M = Fd - F(d - a) = Fa \qquad \text{Eq (29)}$$

The moment of a couple about a point is called the torque of the couple, M or T. The moment of the resultant force about any point on the line of action of the given single force must be of the same sense as that of the couple, positive if the moment of the couple is positive conversely. A couple can be illustrated in following figure.

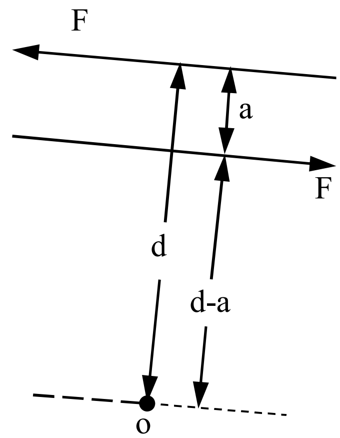


Figure 12 : A couple

f. Equilibrium of Rigid Bodies

A body is in equilibrium when it is stationary or in steady translation relative to an inertial reference frame. In fact, the concept of equilibrium is used for determining unknown forces and moments of forces that act on or within a rigid body or system of rigid bodies. A rigid body is in static equilibrium when the equivalent force-couple system of the external forces acting on it is zero. If the sum of the forces acting on a body is zero and the sum of the moments about one point is zero, then the sum of the moments about every point is zero. In vector notation, this condition is expressed as[8]

$$\sum F = 0 \qquad \text{Eq (30)}$$

$$\sum M_O = \sum (r \times F) = 0 \qquad \text{Eq (31)}$$

In terms of rectangular scalar components, equilibrium can be expressed as

$$\sum F_x = 0 \qquad \sum M_x = 0 \qquad \text{Eq (31)}$$

$$\sum F_y = 0 \qquad \sum M_y = 0 \qquad \text{Eq (32)}$$

$$\sum F_z = 0 \qquad \sum M_z = 0 \qquad \text{Eq (33)}$$

The three couples which may be combined by their moment vectors into a single resultant couple having the moment whose moment vector makes angles. The resultant can be calculated as resultant of any number of forces applied to a rigid body at the same point by replacing force, F with moment, M.

g. Support of Rigid Bodies

According to the number of unknown forces existing, the first step in the solution of problems in statics is the determination of the supporting forces which is defined as the external forces in equilibrium acting upon a body. When the forces are all in either the same or different planes and act at a common point, two or three unknown forces may be determined if their lines of action are known, one if unknown. The following data are required for the complete knowledge of supporting forces: magnitude, direction, and point of application.

As example, the beam has a pin support at the left end A and a roller support at the right end B. The beam is loaded by a force F and a moment M at C as shown in figure 13(a) and 13(b). The steps required to determine the reactions, forces and couples exerted on a body by its supports are[8]:

a. Draw the free-body diagram, isolating the body from its supports and showing the forces and the reactions
b. Apply the equilibrium equations to determine the reactions

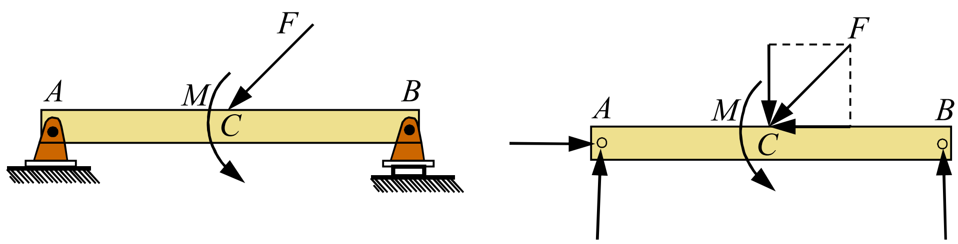


Figure 13 : A beam to determine reaction

The line of action of the forces exerted on the member or by the member at these two points must be along a line connecting the pins, they must meet in a point, when a member of a truss or frame in equilibrium is pinned at two points and loaded at these two points only as shown as follow[6].

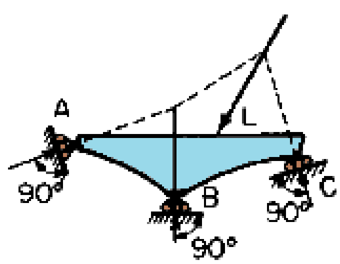


Figure 14 : a bridge to determine reaction

When the conditions are sufficient for the determination of the supports or other forces, the structure is said to be statically determinate; the unknown forces can then be determined from considerations involving the deformation of the material.

When several bodies are so connected to one another as to make up a rigid structure, the forces at the points of connection must be considered as internal forces and are not taken into consideration in the determination of the supporting forces for the structure as a whole.

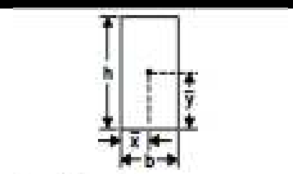
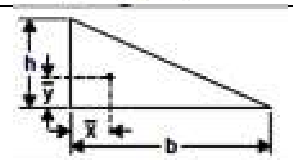
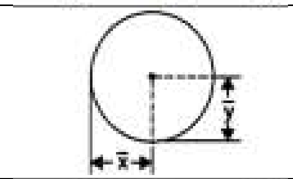
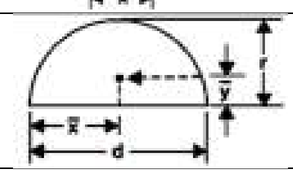
h. Center of Gravity

Center of gravity of the body may be defined as the point through which the whole weight of a body may be assumed to act. The center of gravity of a body is usually denoted by G. Whenever the density of the body is uniform, it will be a constant factor and like geometric shapes of different densities will have the same center of gravity. The term centroid is used in this case since the location of the center of gravity is of geometric concern only. If densities are non-uniform, like geometric shapes will have the same centroid but different centers of gravity. Thus, centroid can be taken as quite analogous to center of gravity when bodies have area only and not weight[1].

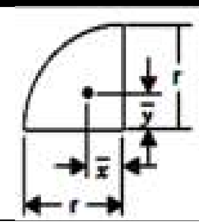
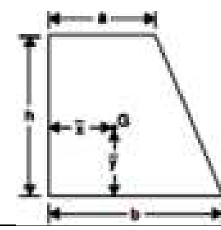
i. Centroid of Plane Area

The position of centroids of some plane geometrical figures can be given in following table.

Table 1 : position of centroids of some plane geometrical

Shape	Area	$\bar{x}$	$\bar{y}$	Figure
Rectangle	bh	$\frac{b}{2}$	$\frac{h}{2}$	
Triangle	$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$	
Circle	$\frac{\pi}{4}d^2$	$\frac{d}{2}$	$\frac{d}{2}$	
Semicircle	$\frac{\pi}{8}d^2$	$\frac{d}{2}$	$\frac{4r}{3\pi}(= 0.424r)$	

Continue from table 1

Shape	Area	$\bar{x}$	$\bar{y}$	Figure
Quadrant	$\frac{\pi}{16}d^2$	$0.424r$	$0.424r$	
Trapezium	$(a+b)\frac{h}{2}$	$\frac{a^2+b^2+ab}{3(a+b)}$	$\frac{(2a+b)}{(a+b)}x\frac{h}{3}$	

j. Centroids of Composite Areas

The location of centroid of a plane can be thought of as the average distance of the area to an axis. In determining the location of the centroid, it follows first moment of area that is found advantageous to place the X-axis through the lowest point and the Y-axis through the left edge of the figure. The figure is divided into simple areas such as rectangles, triangles, etc, then takes the moment of each single area about the Y and X-axis. An example of centroid of a composite area can be illustrated as below.

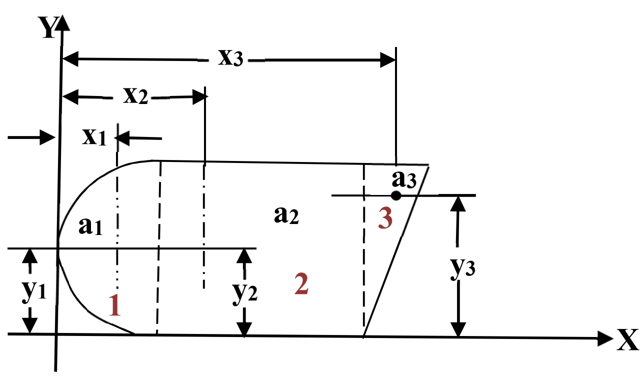


Figure 15 : composite area divided into simple areas

They can be expressed as follows:

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + ... + A_n x_n}{A_1 + A_2 + ... + A_n} = \frac{\sum Ax}{\sum A} \qquad \text{Eq (34)}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + ... + A_n y_n}{A_1 + A_2 + ... + A_n} = \frac{\sum Ay}{\sum A} \qquad \text{Eq (35)}$$

Where,

A = area, ft^2 (m^2)
$\bar{x}$ = centroid coordinate, x
$\bar{y}$ = centroid coordinate, y
x = distance of composite area on x-axis, ft (m)
y = distance of composite area on y-axis, ft (m)

1. Center of Gravity of Simple Solids

The weight of the body is a force acting at its own center of gravity and directed towards the center of the earth. The position of the centers of the bodies weighing, W_1, W_2, W_3, etc. is found in the same manner as the resultant of parallel forces. They can be expressed as follows:

$$\bar{x} = \frac{\sum W_x}{\sum W} = \frac{\sum wV_x}{\sum wV} \qquad \text{Eq (36)}$$

$$\bar{y} = \frac{\sum W_y}{\sum W} = \frac{\sum wV_y}{\sum wV} \qquad \text{Eq (37)}$$

$$\bar{z} = \frac{\sum W_z}{\sum W} = \frac{\sum wV_z}{\sum wV} \qquad \text{Eq (38)}$$

If all bodies are of the same material and weigh w, the equations can be considered by volume only or is called center of volume, and when their bodies are of the constant cross-section, but different lengths (center of their lengths), they become[1]:

$$\bar{x} = \frac{\sum V_x}{\sum V} = \frac{\sum AL_x}{\sum AL} = \frac{\sum L_x}{\sum L} \qquad \text{Eq (39)}$$

$$\bar{y} = \frac{\sum V_y}{\sum V} = \frac{\sum AL_y}{\sum AL} = \frac{\sum L_y}{\sum L} \qquad \text{Eq (40)}$$

$$\bar{z} = \frac{\sum V_z}{\sum V} = \frac{\sum AL_z}{\sum AL} = \frac{\sum L_z}{\sum L} \qquad \text{Eq (41)}$$

Where,

W = weight, lbf (N)
w = weigh, lbf/ft^3 (N/m^3)
V = volume, ft^3 (m^3)
L = length of line, ft (m)

k. Moment of Inertia

The moment of inertia is that property in a rotational system which may be considered equivalent to the mass in a translational system. Generally, the moment of inertia of the body about an axis may be expressed by

$$I = \int y^2 dm \qquad \text{Eq (42)}$$

Where,

I = moment of inertia, lbm.ft^2 (kg.m^2)
y = distance from x-axis, ft (m)
m = mass, lb (kg)

If a body is considered to be composed of a number of parts, its moment of inertia about an axis is equal to the sum of the moments of inertia of the several parts about the same axis.

1. Second Moments

The second moment of area, also known as the area moment of inertia with respect to a given axis is the limit of the sum of the products of the elementary areas into which the area may be conceived to be divided and the square of their distance from the axis in question. It can be expressed as follow:

$$I_{xx} = \int y^2 dA \qquad \text{Eq (43)}$$

$$I_{yy} = \int x^2 dA \qquad \text{Eq (44)}$$

The second moments of the area A about x and y axes is illustrated in following figure.

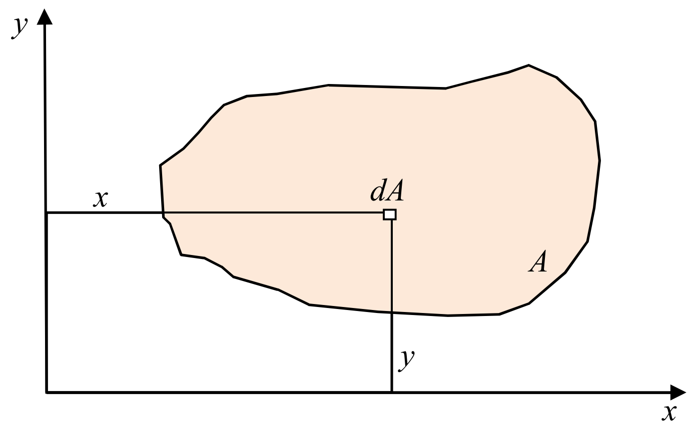


Figure 16: a planar surface of area

The entire area may be concentrated at a single point (k_x, k_y) to give the same moment of area for a given reference. The distance k_x and k_y are called the radii of gyration or the radius of inertia. They are related by Eq (43) and (44), therefore, they become

$$k_x = \sqrt{\frac{\int y^2 dA}{A}} \qquad \text{Eq (45)}$$

$$k_y = \sqrt{\frac{\int x^2 dA}{A}} \qquad \text{Eq (46)}$$

Where,

k = radii of gyration, ft (m)

A = area, ft^2 (m^2)

x = distance from y-axis, ft (m)

y = distance from x-axis, ft (m)

This point (k_x, k_y) depends on the shape of the area and on the position of the reference. The centroid location is independent of the reference position. The moment of inertia of a solid of elementary thickness about an axis is equal to the moment of inertia of the area of one face of the solid about the same axis multiplied by the mass per unit volume of the solid times the elementary thickness of the solid[6].

2. Parallel-Axis Transformations of Moments of Inertia

It is often convenient to first calculate the moment of inertia about a centroidal axis and then transform this with respect to a parallel axis. Moments of inertia are always positive. From parallel axes theorem, it is obvious that the minimum moments of inertia of a body occur about axes that pass through its centroid. The following figures are illustration of parallel axis based on area and mass.

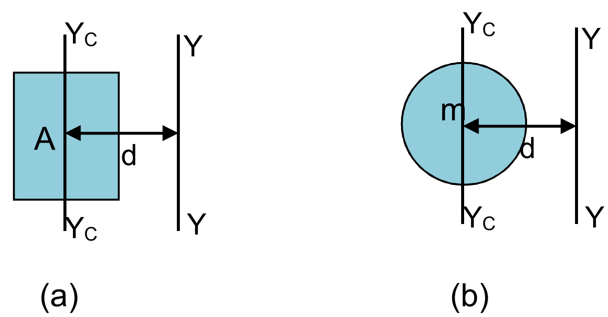


Figure 17 : parallel axis based on area and mass

The formulas for the transformations are defined by [10]

For a mass, m

$$I = I_C + md^2 \quad \text{Eq (47)}$$

For an area, A

$$I = I_C + Ad^2 \quad \text{Eq (48)}$$

$$J_o = J_C + Ad^2 \quad \text{Eq (49)}$$

Where,

I or Jo	=	moment of inertia of m or A about any line, lbm.ft^2 (kg.m^2)
I_C or J_C	=	moment of inertia of m or A about a line through the centroid, lbm.ft^2 (kg.m^2)
d	=	nearest distance between the parallel lines, ft (m)

3. Polar Moment of Inertia

The polar moment of inertia is equal to the sum of the moments of inertia about any two axes at right angles to each other in the plane of the area and intersecting at the pole. It may be described in figure 18.

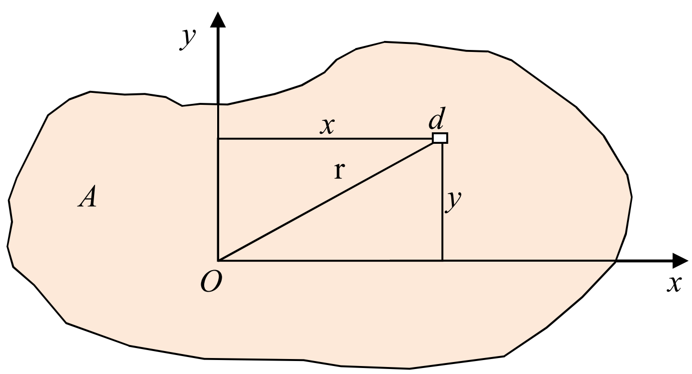


Figure 18 : polar moment of inertia

The sum of second moments of area about orthogonal axes is a function only of the position of the origin O for the axes. Polar moment of inertia can be expressed as follow:

$$I_p = I_{xx} + I_{yy} = \int_A \left(x^2 + y^2\right) dA = \int_A r^2 dA \qquad \text{Eq (50)}$$

4. Product Moment of Inertia

Product of inertia of a body are measures of symmetry. The products of inertia for areas and masses and the corresponding parallel-axis formulas are defined in similar patterns.

This quantity may be positive or negative depending upon the position of the area with respect to the coordinate axes. If the area under consideration has an axis of symmetry, the product of area for this axis and any axis orthogonal to this axis is zero. Product moment of inertia can be defined as follow:

$$I_{xy} = \int xydA \qquad \text{Eq (51)}$$

$$I_{yz} = \int yzdA \qquad \text{Eq (52)}$$

$$I_{xz} = \int xzdA \qquad \text{Eq (53)}$$

Those equations are for area product. They also can used for mass product by replacing area, A to mass, m. Related with the equations, parallel-axis equations become

$$I_{xy} = I_{x'y'} + Ad_x d_y \qquad \text{Eq (54)}$$

$$I_{yz} = I_{y'z'} + Ad_y d_z \qquad \text{Eq (55)}$$

$$I_{xz} = I_{x'z'} + Ad_x d_z \qquad \text{Eq (56)}$$

These transformations are useful for determining the principal moments of inertia which there is at least one pair of rectangular axes in the plane of the area about one of which the moment of inertia is a maximum, and a minimum about the other, and the principal axes when the area or body has no symmetry. The minimum second moment of area corresponds to an axis at right angles to the axis having the maximum second moment. The following figure is illustration of product moment of inertia that is related with parallel-axis.

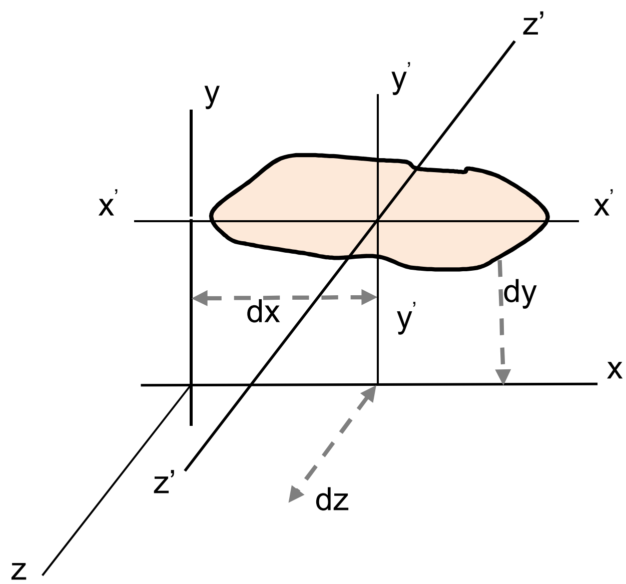
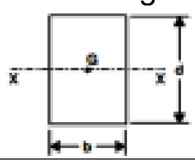
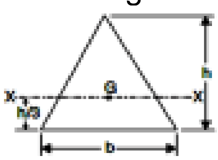
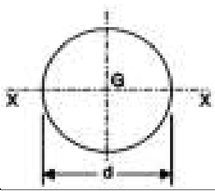
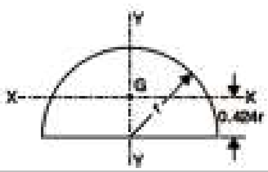
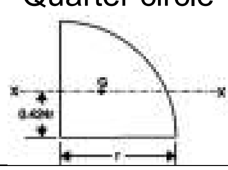


Figure 19 : relation between product of inertia and parallel-axis

To obtain moment of inertia or second moment, there is a table that gives some data on moments of inertia for standard shapes. It can be shown in following table.

Table 2 : moment of inertia

Shape	Area	Centroid	Second Moment of Area, I_{xx} (I_G)
Rectangle 	bh	$y_G = \frac{h}{2}$	$\frac{bh^3}{12}$
Triangle 	$\frac{bh}{2}$	$y_G = \frac{h}{3}$	$\frac{bh^3}{36}$
Circle 	$\frac{\pi d^2}{4}$	$y_G = \frac{d}{2}$	$\frac{\pi d^4}{64}$
Semicircle 	$\frac{\pi d^2}{8}$	$y_G = \frac{2d}{3\pi}$	0.11 r^4
Quarter circle 	$\frac{\pi r^2}{4}$	$y_G = \frac{4r}{3\pi}$	0.055 r^4

II. Fundamental of Solid Mechanic Dynamics

Kinematics of Particles

a. Rectilinear Motion

A body is said to be in motion if it changes its position with respect to its surroundings. The nature of path of displacement of various particles of a body determines the type of motion. The kinematic definitions involve the variables; there are displacement, velocity, and acceleration.

- The displacement of a point is the directed distance that a point has moved on a geometric path from a convenient origin.
- The velocity of a point is its rate of change of its position with respect to its surroundings in a particular direction. It is expressed by

$$v = \frac{ds}{dt} \qquad \text{Eq (57)}$$

- Acceleration of point is the time rate of change of velocity. It is defined by

$$a = \frac{dv}{dt} = v\frac{dv}{ds} \qquad \text{Eq (58)}$$

Where,

a = acceleration, ft/s^2 (m/s^2)
v = velocity, ft/s (m/s)
t = time, second
s = space or distance, ft (m)

In fact, these differential equations are usually limited to the scalar form, since velocity is measured in one dimensional by distance covered per unit time. In addition, it can be expressed in vector quantity, it is measured in two and three dimensional in a particular direction per unit time[1].

i. Space-Time Graphs

A space-time graphs represent the velocity at that time. In figure 20(a), the graph is parallel to the time-axis indicating that the space is not changing with time. The slope of the graph is zero. It means that the body has no velocity and is at rest.

In figure 20(b), The space increases linearly with time. The space increases by equal amounts in equal intervals of time. The slope of the graph is constant. In other words, the body is moving with a uniform velocity.

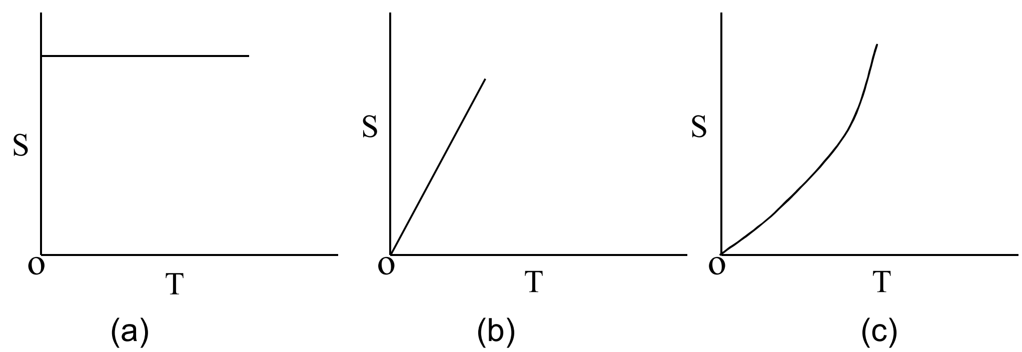


Figure 20 : space-time graph

Refer to Fig. 20 (c). The space-time graph is a curve. This means that the space is not changing by equal amounts in equal intervals of time. The slope of the graph states that the velocity of the body is changing with time. The motion of the body is accelerated.

ii. Velocity-Time Graph

The slope of the graph at any point will represent the acceleration at that time. According figure 21(a), the acceleration of the body is constant. Also, at time t = 0, the velocity is finite. Thus, the body, moving with a finite initial velocity, is having a constant acceleration. The area under the velocity-time curve between any two ordinates represent the distance moved in certain time.

As the time passes, the velocity decreases linearly with time until its final velocity becomes zero, i.e. it comes to rest. Thus, the body has a constant deceleration since the slope of the graph is negative. It is illustrated in figure 21(b). Whereas, refer to figure 21(c), this figure is close with figure 20(c). However, the body does not have a uniform acceleration since the acceleration is changing with time.

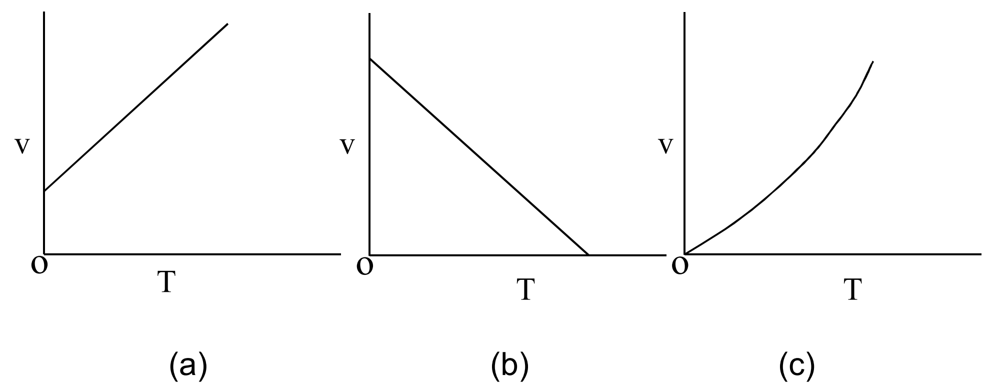


Figure 21 : velocity-time graph

iii. Relations of Equations of Motion for Rectilinear Motion

When the body is moving with a uniform velocity as shown in figure 20(b), the acceleration must be zero. For this motion, space is be given by

$$s = s_0 + v\ t \qquad \text{Eq (59)}$$

Whereas, when the body is moving with a constant acceleration and velocity are not uniform, the equations of motion can be given as follows[7]:

$$v = v_0 + at \qquad \text{Eq (60)}$$

$$v^2 = 2a(s - s_0) + v_0^2 \qquad \text{Eq (61)}$$

$$s = \frac{1}{2}at^2 + v_0 t + s_0$$ Eq (62)

Where,

s_0 = initial space, ft (m)

v_0 = initial velocity, ft/s (m/s)

iv. Resolution of Velocities and Acceleration

A velocity can be obtained by a vector that is the geometric sum of the vectors representing the other two velocities. This velocity, v_r can be resolved by parallelogram of motion as Eq (5) and (14). Resultant acceleration, a_r also can be resolved by using parallelogram of motion as to resolve velocity.

b. Circular and Curvilinear Motion

In a rectilinear motion, a moving particle describes a straight path while in a curvilinear motion it describes a curved path. In circular or rotating motion a moving particle describes a circular path, its position at any instant can be defined by the angle θ covered by X or Y-axis. Similar with rectilinear motion, this motion involves displacement, velocity, and acceleration in angular motion.

– Angular displacement is defined when a particle moves on a circular path its angle of rotation (or its angular displacement) θ, varies with time.

– Angular velocity is defined as the time rate of change of angular displacement. The particle is said to have uniform rotation or uniform angular velocity if at any instant, having covered angular displacement θ in t time, receives equal increment of finite angular displacement. Then, mathematically,

$$\omega = \frac{d\theta}{dt}$$ Eq (63)

For a particle that moves in circular or rotation motion, the instantaneous velocity at that instant can be defined by

$$v = \omega r \qquad \text{Eq (64)}$$

Where,

θ	= angular displacement, rad
t	= time, second or hour
ω	= angular velocity, rad/s
v	= velocity, rad/s
r	= radius of circular path, ft (m)

– Angular acceleration is defined as the rate of change of angular velocity. It can be expressed by

$$\alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta} = d^2\theta dt^2 \qquad \text{Eq (65)}$$

Where,

α	= angular acceleration, rad/sec^2

i. Relations of Equations of Motion for Angular Motion

These relations are similar with rectilinear motion, but the variables are in angular motion. The equations of motion can be given as follows[1]:

$$\omega = \omega_0 + \alpha t \qquad \text{Eq (66)}$$

$$\theta = \frac{(\omega_0 + \omega)}{2t} \qquad \text{Eq (67)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \qquad \text{Eq (68)}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \text{Eq (69)}$$

ii. Curvilinear Motion in a Plane

The motion of the point along a curvilinear path can be specified in terms of its position, velocity, and acceleration vectors. In general, the representations of the position, velocity, and acceleration vectors are different in different coordinate systems. There are several points when considering curvilinear motion. First, the instantaneous velocity vector is always tangent to the path of the particle. Second, the speed of the particle is the magnitude of the velocity vector. Third, the acceleration vector is not tangent to the path of the particle and not collinear with v in curvilinear motion. If the acceleration at some point in the path is resolved by means of a parallelogram into components tangent and normal to the path.

They are shown in figure 22.

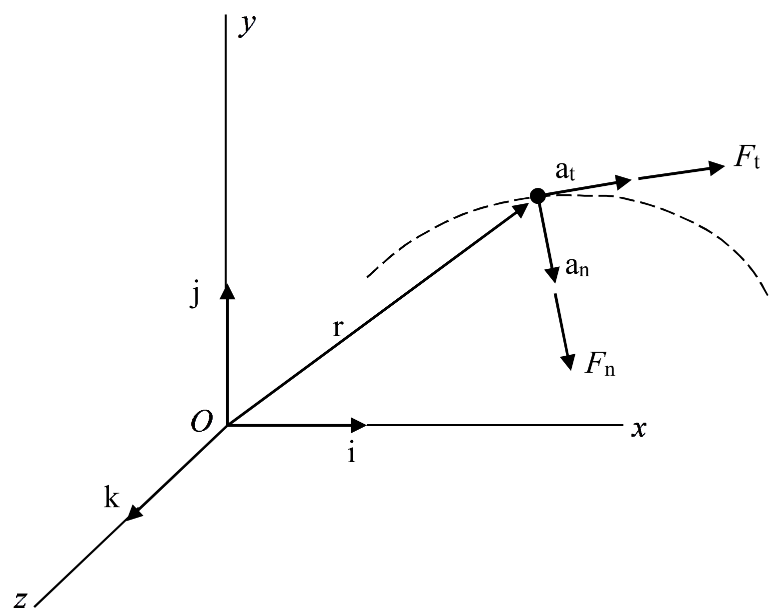


Figure 22 : normal and tangential accelerations

The acceleration of the particle in terms of normal and tangential components can be defined by[6]

$$a = \sqrt{a_n^2 + a_t^2} \qquad \text{Eq (70)}$$

Where,

$$a_t = \frac{dv}{dt} \qquad \text{Eq (71)}$$

$$a_n = \frac{v^2}{\rho} \qquad \text{Eq (72)}$$

Where,

a = total acceleration, rad/s^2
a_t = tangential acceleration, rad/s^2
a_n = normal acceleration, rad/s^2
v = velocity tangent to the path, rad/s
ρ = radius of curvature of the path, ft (m)
(in circular motion, it can be replaced by radius, r)

Motion of a Particle in Polar Coordinates

The motion of the particle may be described in terms of polar coordinates. There are contained two components; radial, r and displacement, θ. The following figure is illustration of polar coordinates.

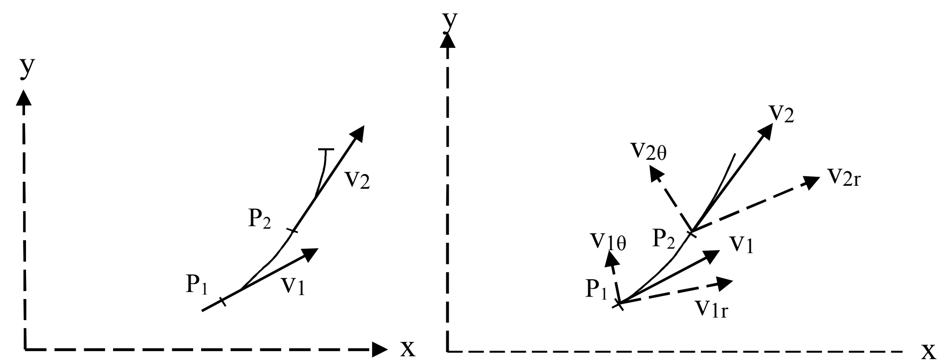


Figure 23 : illustration of polar coordinates

Velocity and acceleration may be expressed in polar coordinates which are defined by

$$v = \sqrt{v_r^2 + v_\theta^2} \quad \text{Eq (73)}$$

$$a = \sqrt{a_r^2 + a_\theta^2} \quad \text{Eq (74)}$$

Where,

$$v_r = \frac{dr}{dt} \quad \text{Eq (75)}$$

$$v_\theta = r\frac{d\theta}{dt} \quad \text{Eq (76)}$$

$$a_r = \frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 \quad \text{Eq (77)}$$

$$a_\theta = r\frac{d^2\theta}{dt^2} + 2\left(\frac{dr}{dt}\right)\left(\frac{d\theta}{dt}\right) \quad \text{Eq (78)}$$

When a point moves on a circular path, velocity can be obtained by eq (64) and the two components of the linear acceleration are expressed by

$$a_n = \frac{v^2}{r} = \omega^2 r = v\omega \qquad \text{Eq (79)}$$

$$a_t = \alpha r \qquad \text{Eq (80)}$$

Kinematics of a Rigid Body

A rigid body is defined as one in which the particles are rigidly connected that does not deform, or change shape. There are three types of rigid body motion which are described as follows.

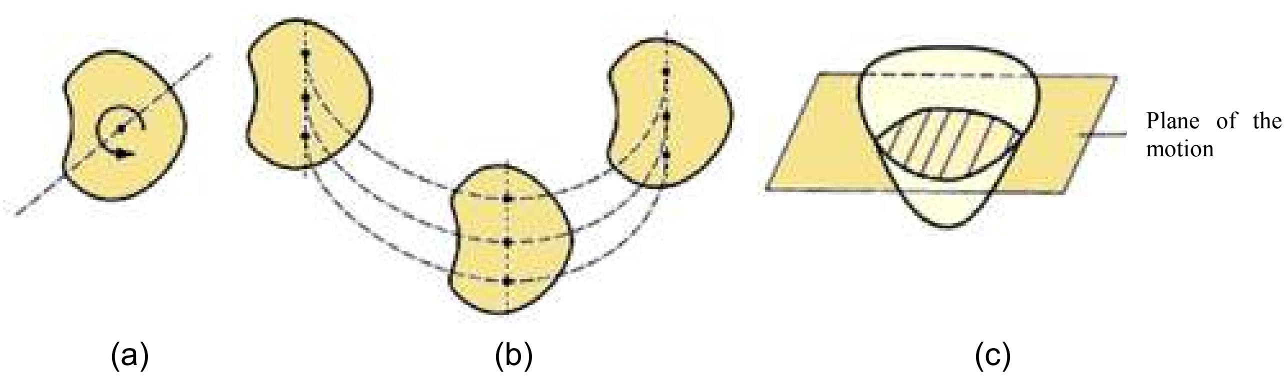


Figure 24 : types of rigid body motion

1. Rotation about a fixed axis

The particles forming the rigid body move in parallel planes along circles centered on the same fixed axis. The particles located on the axis have zero velocity and acceleration, when the axis of rotation intersects the rigid body. Velocity, acceleration and displacement in rotation are measured in angular. This motion is shown in figure 24(a).

2. Translation

Translation occurs when all particles forming the body move along parallel paths. Every point of a rigid body in translation has the same direction, velocity and acceleration during movement. When a rigid body is in translation, the motion of a single point completely specifies the motion of the whole body. Translation is shown in figure 24(b).

3. Planar motion

The points of the rigid body intersected by the plane remain in the plane for two dimensional, or planar motion. The fixed plane is the plane of the motion. Planar motion or complex motion exhibits a simultaneous combination of rotation and translation. This motion is shown in figure 24(c)[8].

a. Motion of a Rigid Body in a Plane

Plane motion is the motion of a rigid body such that the paths of all particles of that rigid body lie on parallel planes. There are several motions of a rigid body that should be considered in a plane.

i. Instantaneous Center of Rotation

The method of instantaneous center of rotation is a geometric method of determining the angular velocity when two or more velocity vectors are known for a given rigid body. Instantaneous center of rotation also can be decided by its axis. When the axis about which any body may be considered to rotate changes its position, any one position is known as an instantaneous axis. The instantaneous axis for the body will be at the intersection of the lines drawn from each vector and perpendicular to its velocity and the line through all positions of the instantaneous axis is called the centrode or instantaneous center. The following figure is illustration for instantaneous center of rotation.

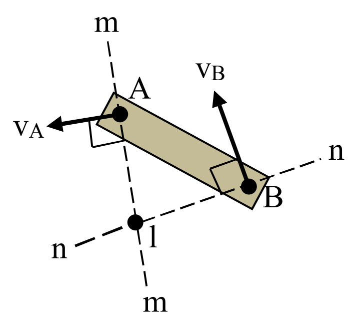


Figure 25 : illustration for instantaneous center of rotation

Based on the figure, the body is rotating about point I at that instant. Point I has zero velocity at that instant, but generally has an acceleration. The angular velocity of the body can be defined by

$$\omega = \frac{v_A}{r_{AI}} = \frac{v_B}{r_{BI}} \qquad \text{Eq (81)}$$

ii. General Motion of a Rigid Body

The general motion of a point moving in a coordinate system which is itself in motion is complicated and can best be summarized by using vector notation. It is important to note that here the angular velocity and angular acceleration vectors are not necessarily in the same direction as they are in general plane motion. The following figure is example to illustrate general motion of a rigid body.

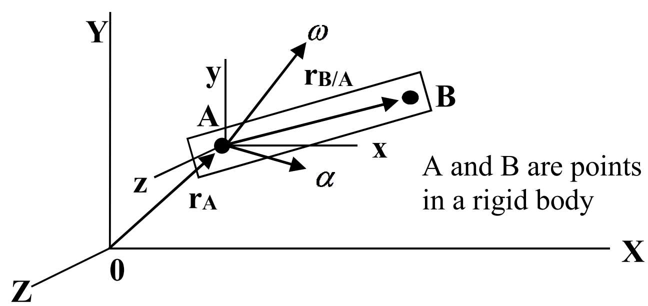


Figure 26 : general motion of a rigid body.

The velocity of a point on the rigid body can be given by[10]

v_B = translation velocity + rotation velocity

$$v_B = v_A + (\omega \times r_{B/A}) \qquad \text{Eq (82)}$$

The acceleration of a point on the rigid body can be given by

a_B = translation acceleration + rotation acceleration

$$a_B = a_A + \left[\left(a_{B/A}\right)_t + \left(a_{B/A}\right)_n\right] \qquad \text{Eq (83)}$$

$$a_B = a_A + \left[\left(\alpha \times r_{B/A}\right) + \left(\omega^2 r_{B/A}\right)\right] \qquad \text{Eq (84)}$$

Dynamics of a Particle

a. Newton’s Second Law of Motion

It states that the rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the straight line in which the force acts. This law enables to measure a force and establish the fundamental equation of dynamics. Newton's second law may be written as

$$\sum F = ma \qquad \text{Eq (85)}$$

When the acceleration due to gravity, it influences the weight of a particle of mass due to the gravitational. Therefore, it can be defined by

$$W = mg \qquad \text{Eq (86)}$$

Where,

- $\sum F$ = total of force, lbf (N)
- m = mass, lbm (kg)
- W = weight, lbf (N)
- g = acceleration gravitional, 32.2 ft/s^2 (9.81 m/s^2)

In rectilinear motion, the acceleration and the direction of the unbalanced force must be in the direction of motion. Forces must be in balance and the acceleration equal to zero in any direction other than the direction of motion.

b. Principle of Work and Energy

When a body is displaced against resistance or accelerated, work must be done upon it. A body is said to possess energy when it can do work. When a body is so held that it can do work and a body is moving with some velocity, it is said to possess energy of motion or kinetic energy. The kinetic energy of a particle of mass with the velocity is expressed as

$$KE = \frac{1}{2}mv^2 \qquad \text{Eq (87)}$$

When a body is released, it is said to possess energy of position or potential energy. Potential energy can be defined by

$$PE = mgh \qquad \text{Eq (88)}$$

If a force which varies acts through a space on a body of mass, the work done is

$$W = \int_{s_1}^{s} F ds \qquad \text{Eq (89)}$$

The work done on a particle as it moves between two positions equals the change in its kinetic energy. It is called the principle of work and energy that may be expressed as

$$KE = \frac{1}{2} m \left(v_2^2 - v_1^2 \right) \qquad \text{Eq (90)}$$

i. Work

Work is measured by the product of force and displacement both being in the same direction. Work is positive or negative, according as the force acts in the same direction or in the direction opposite to the direction of displacement. The work is given by

$$W = \int_{s_1}^{s_2} F ds \qquad \text{Eq (91)}$$

The result of integration,

$$W = F \times s \qquad \text{Eq (92)}$$

ii. Power

Power is defined as rate of doing work. Power can be given by

$$P = \frac{W}{t} = F.v \qquad \text{Eq (93)}$$

Where,

KE	=	kinetic energy, J (N.m)
PE	=	potential energy, J (N.m)
W	=	Work, J (N.m)
P	=	Power, HP (watt)
s	=	space or displacement, ft (m)
h	=	height above the ground, ft (m)

c. Conservation of Energy

Conservation of energy means that the sum of changement of the kinetic energy and the potential energy. It can be expressed as follow.

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2 \qquad \text{Eq (94)}$$

d. Principle of Impulse and momentum

Impulse is the product of the force and the time that force acts[9]. This concept is merely an outgrowth of Newton's second law and combined by acceleration definition. It results the momentum that means the product of mass and velocity. It includes vector quantity. Impulse can be defined by

$$F = m \cdot a \qquad \text{Eq (95)}$$

$$F = m \cdot \frac{dv}{dt} \qquad \text{Eq (96)}$$

The result of integral,

$$F(t_2 - t_1) = m(v_2 - v_1) \qquad \text{Eq (97)}$$

This principle is defined that the impulse applied to a particle during an interval of time is equal to the change in its linear momentum.

e. Conservation of Momentum

The law of conservation of momentum states that total momentum of any group of objects always remains the same if no external force acts on them. It means that the total momentum before collision = total momentum after collision[1]. In case, there are two particles A and B with the velocities v_{A1} and v_{B1} collide. The velocities of A and B after the impact are v_{A2} and v_{B2}. Conservation of momentum is illustrated as follow[8].

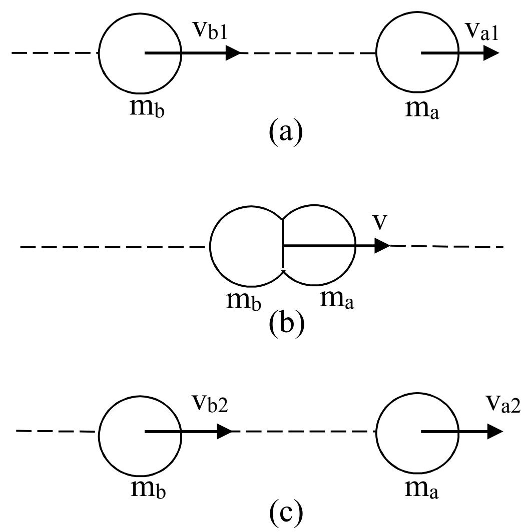


Figure 27 : impact

$$F_A + F_B = 0 \quad \text{Eq (98)}$$

$$m_A \cdot a_A = m_B \cdot a_B \quad \text{Eq (99)}$$

$$m_A \cdot \frac{dv_A}{dt} = m_B \cdot \frac{dv_B}{dt} \quad \text{Eq (100)}$$

$$m_A \cdot v_{A_2} - m_A \cdot v_{A_1} = m_B \cdot v_{B_2} - m_B \cdot v_{B_1} \quad \text{Eq (101)}$$

Therefore, conservation of momentum can be obtained by

$$m_A \cdot v_{A_2} + m_B \cdot v_{B_2} = m_A \cdot v_{A_1} + m_B \cdot v_{B_1} \quad \text{Eq (102)}$$

Where,

v_{A1} = velocity of particle A before collision, ft/s (m/s)
v_{A2} = velocity of particle A after collision, ft/s (m/s)
v_{B1} = velocity of particle B before collision, ft/s (m/s)
v_{B2} = velocity of particle B after collision, ft/s (m/s)

i. Impact

The following deals with the impact of elastic and inelastic spheres, although it applies to bodies of any shape. When considering the response of two deformable bodies to direct central impact, the coefficient of restitution is used. This coefficient *e* can be given as

$$e = -\frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}} \quad \text{Eq (103)}$$

For real materials, $0 < e < 1$. If both bodies are perfectly elastic, $e = 1$, and if either body is perfectly plastic, $e = 0$ and v_{A2} is similar with v_{B2}.

f. Principle of Angular Impulse and Momentum

The method of angular momentum is based on the momentum of a particle about a fixed point, using the vector cross product in the general case of Newton's second law with the position vector r as illustrated as follow.

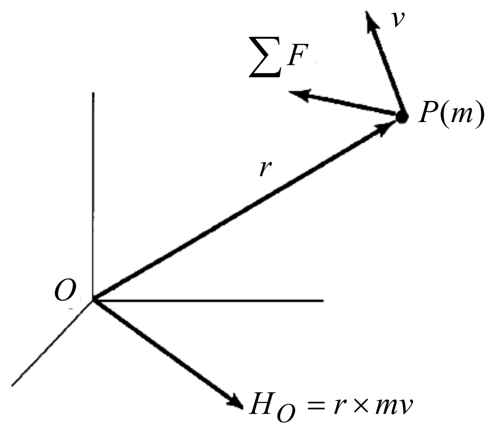


Figure 28 : angular impulse and momentum

The equation that can be given,

$$r \times F = r \times ma = r \times m\frac{dv}{dt}$$ Eq (104)

After it undergoes time derivative, the equation becomes,

$$r \times F = \frac{dH_O}{dt}$$ Eq (105)

where the vector,

$$H_O = r \times mv$$ Eq (106)

The angular momentum equation can be solved using a scalar method if the motion of the particle remains in a plane[10],

$$H_O = mr^2\omega$$ Eq (107)

Where,

H_O = angular momentum about O, ($kg.m^2/s$)

rxF = rate of change of the moment of momentum about point O,
r = distance between point O and P, ft (m)

Dynamics of Rigid Bodies

The fundamental equation for rigid body should consider translation and rotation motion. The equations for dynamics of rigid bodies can be explained as follow.

a. Equation of Motion for the Center of Mass

The equation for translation is the sum of external forces in two or three dimensions which is based on Newton's second law as Eq (85) using acceleration of the center of mass. Whereas, the equation for rotation states that the the sum of the external moments on the rigid body as given by

$$\sum M_C = I_C \alpha$$ Eq (108)

Where,
M_C =moments, lbf.ft (Nm)
I_C = moment of inertia about the center of mass, lbm.ft^2 (kg.m^2)
α = angular acceleration of the body about the center of mass, rad/sec^2

b. Rotation about a Fixed Axis Not Through the Center of Mass

If the axis of rotation does not pass through the center of gravity, the center of gravity will have the resultant acceleration and force acting on the body must also have normal and tangential components. The mass of the rotating body may be nonuniformly distributed as illustrated as follow.

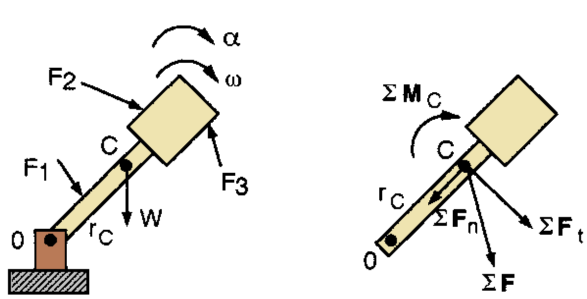


Figure 29 : rotation of a rigid body about a fixed axis

The equations which can be given from figure, are

$$\sum F_n = mr_C\omega^2 \qquad \text{Eq (109)}$$

$$\sum F_t = mr_C\alpha \qquad \text{Eq (110)}$$

$$\sum M_o = I_o\alpha \qquad \text{Eq (111)}$$

Where,

r_C = distance between the fixed axis O and the mass center C, ft (m)

c. General Plane Motion

A body that is translating and rotating is in general plane motion. For any arbitrary point P, moving or fixed, the scalar equations of motion with vector cross product are given by

$$\sum M_P = I_P\alpha + r \times ma_C \qquad \text{Eq (112)}$$

d. Work and Energy Methods for Rigid Bodies in Plane Motion

a. Work Methods for Rigids Body

As for particles, the work of a force acting on a rigid body is defined by

$$W = \int_{r_1}^{r_2} Fdr = \int_{r_1}^{r_2} F \cdot vdt \qquad \text{Eq (113)}$$

The work of a moment can be defined by the increment of work done by a couple M acting in a body during an increment of angular rotation dθ in the plane of the couple.

$$W = \int_{\theta_1}^{\theta_2} M \cdot d\theta \qquad \text{Eq (114)}$$

b. Kinetic Energy of a Rigid Body

The kinetic energy is the algebraic sum of the translating kinetic energy of the center of gravity and the rotating kinetic energy about the center of gravity. It can be defined by

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad \text{Eq (115)}$$

c. Work – Energy Equation

The principle of work and energy for a rigid body is the same as used for particles with the addition of the rotational energy terms.

$$KE_2 = KE_1 + W \qquad \text{Eq (116)}$$

The conservation of energy in a conservative rigid body system is given as follow:

$$KE1 + PE_{g1} + PE_{e1} = KE_2 + PE_{g2} + PE_{e2} \qquad \text{Eq (117)}$$

Where,

KE_1 = initial kinetic energy of the body, btu (J)
KE_2 = final kinetic energy of the body, btu (J)
PE_g = potential energy gravitional of body, btu (J)
PE_e = potential energy elastic of body, btu (J)

e. Impulse and Momentum of a Rigid Body

Impulse and momentum methods are particularly useful when time and velocities are of interest. They can be considered by several variables which are described in figure 30.

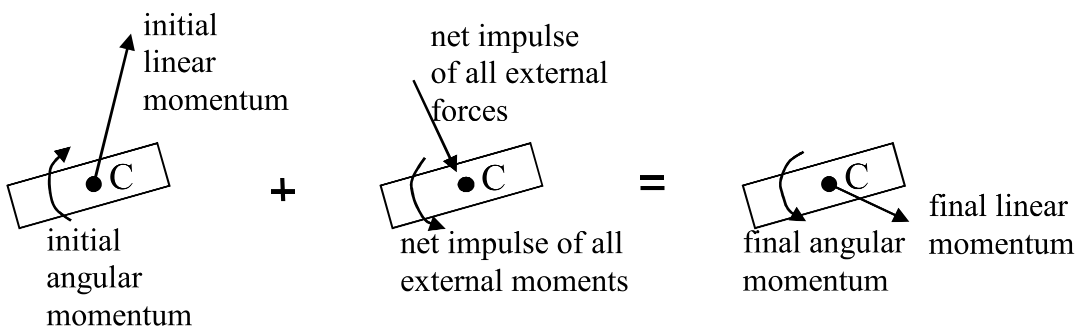


Figure 30 : Impulse and Momentum of a Rigid Body

a. The Angular Momentum of a Rigid Body

The angular momentum of a rigid body in plane motion is the vector sum of the angular momentum about the reference axis and the moment of the linear momentum of the center of gravity about the reference axis.

$$H_C = I_C \omega + r \times mv \qquad \text{Eq (118)}$$

b. The impulse of the external forces and external moments

The impulse of the external forces equation is given by

$$\int_{t_1}^{t_2} \sum F dt = m_{C_2}\left(v_{C_2} - v_{C_1}\right) \qquad \text{Eq (119)}$$

The impulse of the external moments equation is given by

$$\int_{t_1}^{t_2} \sum M_C dt = H_{C_2} - H_{C_1} \qquad \text{Eq (120)}$$

When ∑M is parallel to angular velocity, ω,

$$\int_{t_1}^{t_2} \sum M_C \, dt = I_C \left(\omega_2 - \omega_1 \right) \qquad \text{Eq (121)}$$

f. D' Alembert's Principle

This principle states that on the lines of equation of static equilibrium, equation of dynamic equilibrium can also be established by introducing inertia force in the direction opposite the acceleration in addition to the real forces on the plane. Static equilibrium equations which typically are used,

$$\sum Fx = 0, \qquad \sum Fy = 0, \qquad \sum M = 0 \qquad \text{Eq (122)}$$

Similarly when different external forces act on a system in motion, the algebraic sum of all the forces (including the inertia force) is zero. Newton's second law becomes,

$$F + (-ma) = 0 \qquad \text{Eq (123)}$$

The expression in the bracket (– ma) is the inertia force and negative sign signifies that it acts in a direction opposite to that of acceleration[1].

Friction

Friction is the resistance that is encountered when two solid surfaces slide or tend to slide over each other. There is a force which acts tangentially to the surfaces so as to oppose motion. The surfaces may be either dry or lubricated. Dry friction occurs when the surfaces are free from contaminating fluids, or films, the resistance. Whereas boundary (or greasy) lubrication occurs the rubbing surfaces are separated from each other by a very thin film of lubricant[6]. The following figure is free-body diagram of occurred friction on a block.

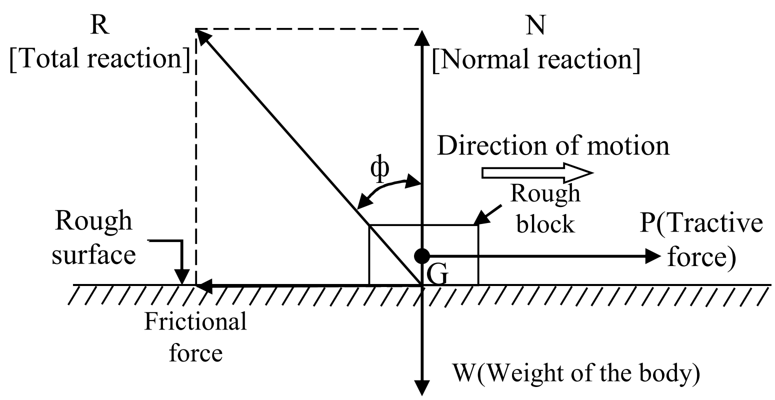


Figure 31 : illustration of occurred friction on a block

a. Static and Dynamic Friction

Static friction is the friction offered by the surfaces subjected to external forces until there is no motion between them. Whereas, the dynamic friction is the friction experienced by a body when it is in motion. The magnitude of friction force depends on the normal force, N pressing the bodies together and on the material along with surface properties which are lumped together and represented by the coefficient of friction μ.

i. Static and Kinetic Coefficients of Friction

There are two values of friction coefficient, the static friction coefficient, μ_s when motion is about to commence or not occur (at rest), and the kinetic friction coefficient, μ_k which is smaller, when there is motion. The magnitude of the friction force, F_f for each friction coefficients can be obtained by

$$F_f = N \tan\theta = N\mu \qquad \text{Eq (124)}$$

Where,

F_f = friction force, lbf (N)

N = normal force, lbf (N)

θ = angle of friction, degree

μ = friction coefficient

μ_s = static friction coefficient

μ_k = kinetic friction coefficient (during sliding)

When $F_f < N\mu_s$, the body is at rest, whereas $F_f > N\mu_s$, motion is occurring[10]. Typical values of friction coefficient for various materials are shown in Table 3.

Table 3 : Values of friction coefficient for various materials[6]

Material	Static, μ_s		Sliding, μ_k	
	Dry	Greasy	Dry	Greasy
Hard steel on hard steel	0.78	0.11 (Oleic acid)	0.42	0.029 (stearic acid)
Hard steel on babbitt (ASTM No. 1)	0.70	0.23 (Atlantic spindle oil)	0.33	0.16 (Atlantic spindle oil)
Mild steel on lead	0.95	0.5 (medium mineral oil)	0.95	0.3 (medium mineral oil)
Copper on mild steel	0.53	0.36	-	0.18 (Oleic acid)
Cast iron on cast iron	1.10	-	0.15	0.070 (lard oil)

b. Friction on an Inclined Plane

In this case, the body is located on an inclined plane under the influence of a force P and weight of body, W. There are considered in motion up and down manner to determine value of force P on an inclined plane.

Following figure is free-body diagram of motion up an inclined plane[1].

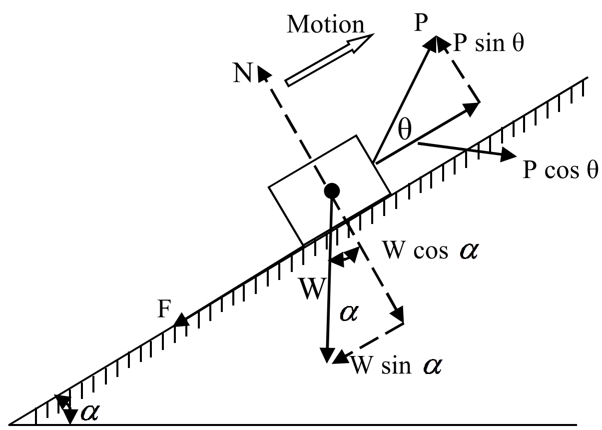


Figure 32 : free-body diagram of motion up an inclined plane

The steps which should be done by construct the free-body diagram, then, resolving the forces parallel and forces perpendicular to the plane. After that, substituting forces perpendicular to the forces parallel for obtaining value of force P. Thus, equations can be given in several conditions by

For motion up on an inclined plane,

$$P = \frac{W \sin(\alpha + \theta)}{\cos(\theta - \theta)} \qquad \text{Eq (125)}$$

When the force P is parallel to the plane, it becomes,

$$P = \frac{W \sin(\alpha + \theta)}{\cos \theta} \qquad \text{Eq (126)}$$

When there is no force of friction,

$$P = \frac{W \sin \alpha}{\cos \theta} \qquad \text{Eq (127)}$$

For motion down on an inclined plane, the value force P can be obtained by similar ways and equations with motion up. However, there is opposite signs in motion down on a plane; positive (+) becomes negative (-) and inversely.

c. Rolling Friction

Rolling friction is the force resisting the motion when a body (such a ball, tire, or wheel) rolls on a surface. The following figure is illustration of rolling friction[7].

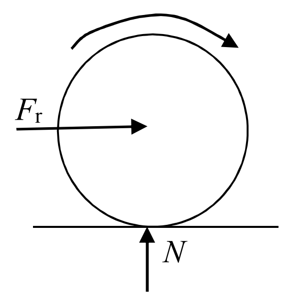


Figure 33 : illustration of rolling friction

The force to move a wheeled vehicle can be given by

$$F_r = \mu_r N \qquad \text{Eq (128)}$$

Where,

F_r = force to move a wheeled vehicle, lbf (N)
μ_r = rolling coefficient of resistance, dimensionless
N = normal force or wheel reaction, lbf (N)

d. The Wedge

A wedge may be used to raise or lower a body. Thus, two directions of motion must be considered in each situation, with the friction forces always opposing the impending or actual motion. To determine value of forces can be used by Lami's theorem as Eq (17). The following figure is illustration of wedge friction.

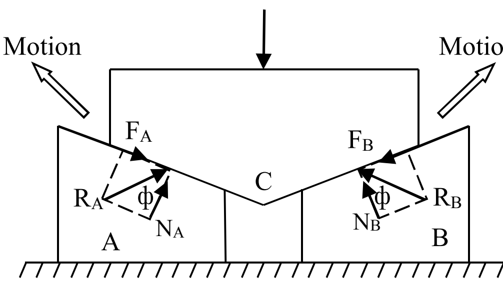


Figure 34 : free-body diagram of wedge

Vibration

Vibrations in machines and structures should be analyzed and controlled if they have undesirable effects such as noise, unpleasant motions, or fatigue damage with potentially catastrophic consequences.

a. Definition of Simple Harmonic Motion

The term harmonic means that the motion is exactly repeated after a certain period of time T. This simple harmonic motion can be represented by a point P on the circumference of circle of radius r as shown as follow.

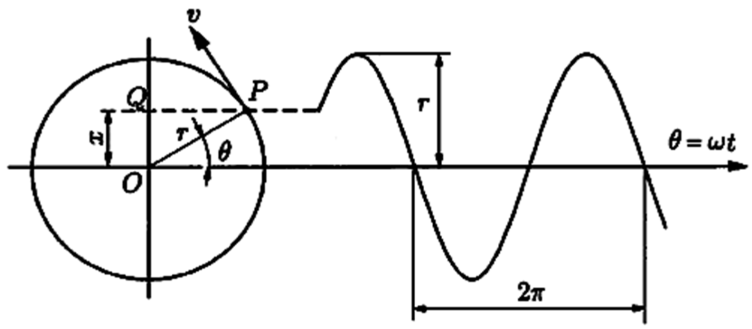


Figure 35 : Simple harmonic motion

The period T for one complete revolution of the point P is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\omega r} = \frac{2\pi}{\omega} \quad \text{Eq (129)}$$

The frequency f is the number of complete oscillations that take place in one second. It is given by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \text{Eq (130)}$$

Where,

T = period, sec/cycle
f = frequency, Hertz (rad/s)

The projection of P on the vertical diameter is the point Q, which moves up and down with simple harmonic motion, thus the displacement x can be given by

$$x = r\sin\theta = r\sin\omega t \quad \text{Eq (131)}$$

The velocity and acceleration of harmonic motion can be determined by

$$v = \dot{x} = \omega r\cos\omega t = \omega r\sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{Eq (132)}$$

$$a = \ddot{x} = -\omega^2 r\sin\omega t = \omega^2 r\sin(\omega t + \pi) \quad \text{Eq (133)}$$

b. Free Undamped Vibration

Free undamped vibration means that the spring has no mass and that there is no damping in the system. The particle is attached to one end of a massless spring of elastic constant k as shown in figure 36.

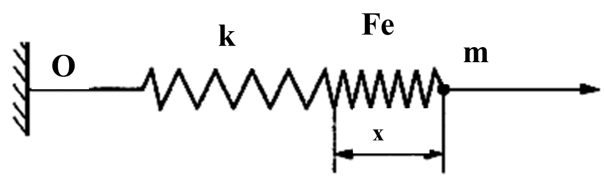


Figure 36 : free undamped vibration

The equation of motion for free vibration (motion under internal forces only; F = 0) can be given by

$$m\ddot{x} + kx = 0 \qquad \text{Eq (134)}$$

It also can become,

$$\ddot{x} + \omega^2 x = 0 \qquad \text{Eq (135)}$$

Where,

ω = $\sqrt{\frac{k}{m}}$ natural circular (angular) frequency, Hz (rad/s)

The displacement *x* as a function of time *t* is given as follow

$$x = C_1 \sin \omega t + C_2 \cos \omega t \qquad \text{Eq (136)}$$

It also can be written as follow

$$x = A\sin(\omega t + \varphi) \qquad \text{Eq (137)}$$

with

$$A = \sqrt{C_1^2 + C_2^2}\ ,\ \tan\varphi = \frac{C_1}{C_2} \qquad \text{Eq (138)}$$

Where,

A = amplitude
φ = phase angle
C = constant

The period of vibration is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad \text{Eq (139)}$$

and the frequency of motion is given by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{Eq (140)}$$

c. Forced Undamped Vibrations

Common sources of harmonic excitation imbalance in rotating machines, the motion of the machine itself, or forces produced by reciprocating machines. Forced undamped vibration can be illustrated as follow.

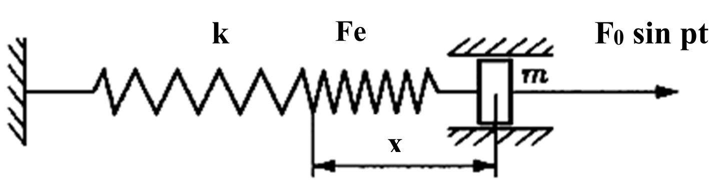


Figure 37 : forced undamped vibration

The equation of motion for a harmonic force of forcing frequency can be defined by combining equation of motion of free vibration.

$$m\ddot{x} + kx = F_0 \sin pt \qquad \text{Eq (141)}$$

The general solution of the forced vibration in the steady state can be defined by

$$x = A \sin pt \qquad \text{Eq (142)}$$

Where,

$$A = \frac{F_0}{k - mp^2} = \frac{F_0 / k}{1 - (p/\omega)^2} \qquad \text{Eq (143)}$$

Where,

F_0 = amplitude of the forced vibration

p = forced angular frequencies, Hz (rad/s)

1. Resonance

The resonance phenomenon appears when the frequency of perturbation or forced angular frequency, p is equal to the natural angular frequency ω. The resonance is characteristic through increasing amplitude to infinity. Dangerously large amplitudes may occur at resonance. A magnification factor is defined as

$$MF = \frac{F}{F_0 / k} = \frac{A}{d_0} = \frac{1}{1 - (p/\omega)^2} \qquad \text{Eq (144)}$$

Where,

F = exciting force, lbf (N)

d_o = amplitude of vibration of the moving support mass

Curve of resonance can be shown in figure 38.

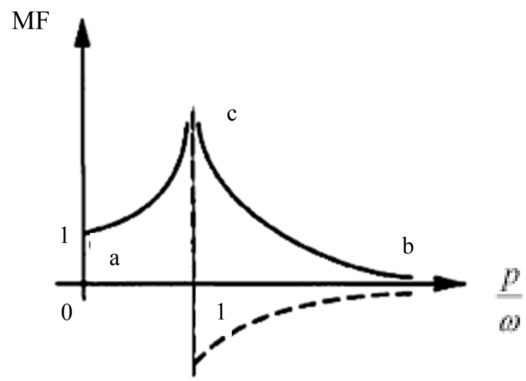


Figure 38 : curve of resonance

Based on the figure, several special cases of this are noted :

1. Static loading at point a ($p = 0$, $MF = 1$), or it states the system vibrates in phase with force ($p<\omega$).
2. Resonance at point c ($p = \omega$, $MF = \infty$)
3. High-frequency excitation b ($p = \infty$, $MF = 0$), or it states the influence of forced force is practically null ($p>\omega$).

d. Free Damped Vibration

All real systems dissipate energy when they vibrate, and the damping must be included in analysis, particularly when the amplitude of vibration is required. This type of vibration appears for the case of motion in a liquid environment with low viscosity. The following figure is illustration for a free vibration with linear damping.

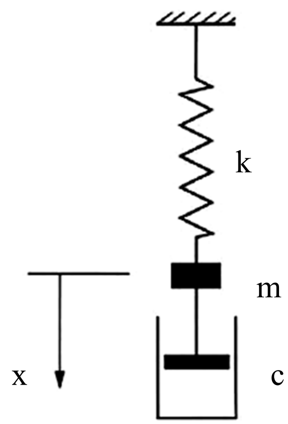


Figure 39 : illustration of free damped vibration

The equation of motion for damped free vibrations can be defined by

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad \text{Eq (145)}$$

Where,

c = coefficient of viscous damping, N.s/m

The characteristic equation in r for Eq (145) can be defined as

$$r_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \qquad \text{Eq (146)}$$

The value of the coefficient of viscous damping, c that makes the radical zero is the critical damping coefficient as given as follow.

$$c_{cr} = 2m\sqrt{\frac{k}{m}} = 2m\omega \qquad \text{Eq (147)}$$

There are several classifications of vibration with respect to roots r and critical damping coefficient as follows:

1. When the roots r_1 and r_2 are complex numbers, it is defined as underdamped system, $c < c_{cr}$. The displacement is defined by

$$x = Ae^{-(c/2m)t}\sin(\omega_d t + \phi) \quad \text{Eq (148)}$$

Which,

$$\omega_d = \omega\sqrt{1-\left(\frac{c}{c_{cr}}\right)^2} \quad \text{Eq (149)}$$

Where,

A	= constant depending on the initial condition
c	= coefficient of viscous damping, Ns/m
c_{cr}	= critical damping coefficient, Ns/m
ω_d	= damped natural frequency, Hz (rad/s)

2. When the roots r_1 and r_2 are real and distinct, it is defined as overdamped system, $c>c_{cr}$ and the motion is nonvibratory or aperiodic. The displacement of this case is[8]

$$x = C_1e^{r_1t} + C_2e^{r_2t} \quad \text{Eq (150)}$$

Where,

C = constant

3. When the roots r_1 and r_2 are real and identical, $r_1 = r_2$, it is defined as critically damped system, $c = c_{cr}$. This motion is also nonvibratory; x decreases at the fastest rate possible without oscillation of the mass.

   The displacement of this case is

$$x = e^{r_{1,2}t}(C_1t + C_2) \quad \text{Eq (151)}$$

The damping in a system is determined by measuring the rate of decay of free oscillations. This is expressed by the logarithmic decrement δ, involving any two successive amplitudes x_i and x_{i+1}.

$$\delta = \ln \frac{x_i}{x_{i+1}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad \text{Eq (152)}$$

Where,

ζ = damping factor, ratio c/c_{cr}

The period of the damped vibration can be given as follow.

$$T_d = \frac{2\pi}{\omega_d} \quad \text{Eq (153)}$$

Where,

T_d = period of the damped vibration, sec/cycle

e. Forced Damped Vibrations

In this case, a mass is attached to a helical spring and is suspended from a fixed support. In additional, the mass is subjected to an oscillating force. Following figure is illustration of forced damped vibration.

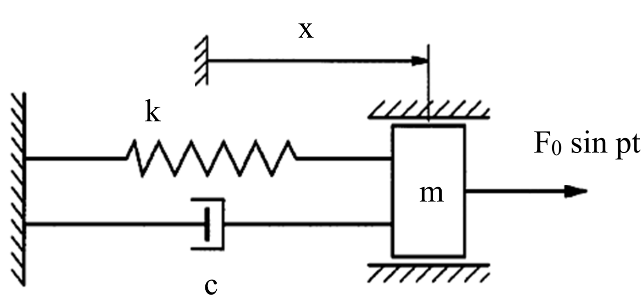


Figure 40 : forced damped vibration

The form of the solution can be simplified by using the technique of the forced vibration response can be written as

$$x = A\sin(pt - \phi)$$ Eq (154)

Where,

$$A = \frac{F_0}{\sqrt{(cp)^2 + (k - mp^2)^2}}$$ Eq (155)

and phase angle can be defined by

$$\tan\phi = \frac{cp}{k - mp^2}$$ Eq (156)

The magnification factor for the amplitude of the oscillations is defined as

$$MF = \frac{A}{F_0 / k} = \frac{A}{d_0} = \frac{1}{\sqrt{[2\zeta(p/\omega)]^2 + [1 - (p/\omega)^2]^2}}$$ Eq (157)

The magnification factor depends upon ζ and p/ω. The magnification factor is plotted versus the ratio p/ω as shown in figure 41.

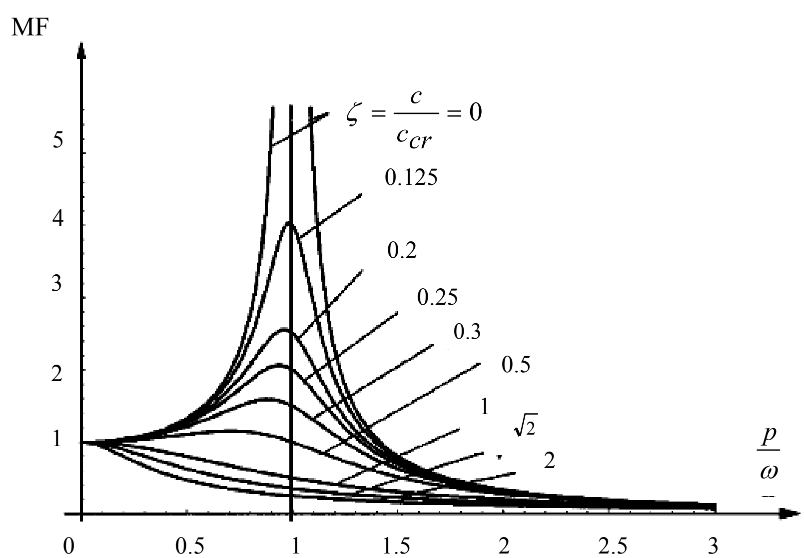


Figure 41 : magnification factor in damped forced vibration

B. Fluid Mechanics

I. Fluid Properties

The important properties of a fluid in fluid mechanics terms are viscosity, density, specific gravity, specific weight and, all of which are intensive properties.

a. Viscosity

Viscosity is a measure of the resistance fluids to flow and may be considered as internal friction. The higher the coefficient of viscosity, the greater the resistance. In laminar flow the shear force between adjacent layers parallel to the direction of flow is proportional to the velocity gradient. Relation between shear force and viscosity can be shown by Eq (158) and then viscosity is obtained by Eq (159).

$$\tau = \frac{F}{A} = \mu \frac{dv}{dy} \qquad \text{Eq (158)}$$

$$\mu = \frac{F}{A}\frac{dy}{dv} \qquad \text{Eq (159)}$$

Where,

τ = shear force, lb/ft.s^2 (N/m^2)
μ = viscosity, lbf.s/ft^2 or poise (N.s/m^2)
F = force, lb.ft/s^2 (N)
A = area, ft^2 (m^2)
dv = velocity differential, ft/s (m/s)
dy = distance differential, ft (m)

In fact, viscosity is divided to be dynamic and kinematic viscosity. Dynamic viscosity is referred to as viscosity. Whereas, kinematic viscosity is a ratio of the viscous force to inertial force. Inertial force is characterized by fluid density. Kinematic viscosity is mathematically defined as follows.

$$\nu = \frac{\mu}{\rho} \qquad \text{Eq (160)}$$

Where,

ν = kinematic viscosity, ft^2/s or stoke (m^2/s)
μ = dynamic viscosity, lbf.s/ft^2 or poise (N.s/m^2)
ρ = fluid density, lb/ft^3 (kg/m^3)

In general the viscosity of liquid mixture has to be evaluated experimentally. Even for homogeneous mixture, there is not silver bullet to estimate the viscosity. For most cases, the following the Heming-Ziperer correlation for the calculation of gas mixture viscosity at low pressure can be defined by[5]:

$$\mu_m = \frac{\sum MF_i \mu_i \left(MW_i\right)^{0,5}}{\sum MF_i \left(MW_i\right)^{0,5}} \qquad \text{Eq (161)}$$

Where,

μ_m = gas mixture viscosity, micropoise (N.s/m^2)
MF_i = mole fraction of component *i*
μ_i = gas viscosity of component *i*, micropoise (N.s/m^2)
MW_i = molecular weight of component *i*, lb/ft^3 (kg/m^3)

Viscosity has several important characteristics. The characteristics are primarily a temperature function and essentially independent of pressure. Viscosity of liquids decreases with temperature whereas viscosity of gases increases with increase in temperature. In addition, viscosity of liquid is due to cohesion and molecular momentum transfer.

b. Density and Specific Gravity

The density is characteristic of each substance. Density is influenced by temperature and pressure that can be proven using gas ideal law as shown in Eq (163). The density of both liquids and gases decreases with increasing temperature, and the density of gases is much more sensitive to temperature than that of liquids. In simple, density of a fluid is its mass per unit volume that expressed in Eq (162).

$$\rho = \frac{m}{V} \qquad \text{Eq (162)}$$

$$\rho = \frac{P \cdot MW}{RT} \quad \text{Eq (163)}$$

Where,

ρ = fluid density, lb/ft^3 (kg/m^3)
m = mass of fluid, lb (kg)
V = volume of fluid, ft^3 (m^3)
P = pressure, lbf/ft^{2}a (N/m^2)
MW = molecular weight, lb/lbmol (kg/kmol)
R = gas law constant, 10.731 ft^3.lb$_f$/in^2.lbmol ^{o}R (8314.34 kg.m^2/s^2.kgmol.K)
T = temperature, ^{o}R (K)

The specific gravity (sp. gr.) of a substance is a dimensionless ratio of the density of a fluid to that of a reference fluid. Since the density of liquids changes with temperature for a precise definition of specific gravity, the temperature of the fluid and the reference fluid should be stated, for example, 60/60°F, where the upper temperature pertains to the liquid and the lower to water. If no temperatures are stated, reference is made to water at its maximum density, which occurs at 3.98°C and atmospheric pressure[6].

c. Specific Weight

The specific weight of a fluid is its weight per unit volume. Unlike density, specific weight is not absolute. It depends upon the value of the gravitational acceleration, which varies with location.

$$\gamma = \rho \cdot g \quad \text{Eq (164)}$$

Where,

γ = specific weight, lbf/ft^3 (N/m^3)
ρ = fluid density, lbm/ft^3 (kg/m^3)
g = acceleration of gravity 32.17 ft/s^2 (9.81 m/s^2)

II. Fluid statics

a. The Basic Equation of Fluid Statics

The basic equation of fluid statics is obtained by consideration of a fluid particle at rest with respect to other fluid particles. For fluids in equilibrium the only internal stresses acting will be normal forces, since the shear stresses depend on velocity gradients, and all such gradients, by the definition of equilibrium, are zero. An equilibrium of forces on an elementary volume element dx, dy, dz; the forces being pressures acting on the faces of the element and gravity acting in the z direction. Thus, it can be expressed in Eq (165).

$$dp = -\rho \cdot g \cdot dz = \gamma \cdot dz \quad \text{Eq (165)}$$

Most liquids are assumed to be of constant density. In such a liquid, difference in pressure is given by integrating Eq (165) and it is illustrated in figure 42.

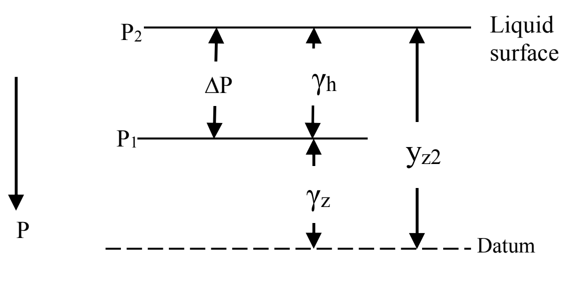


Figure 42 : pressure equivalence

Result of integration, it is also called hydrostatic equation as shown as follows :

$$p_2 - p_1 = -\rho \cdot g(z_2 - z_1) = \gamma \cdot h \quad \text{Eq (166)}$$

Where,

- z = elevation or depth, ft (m)
- p = pressure, lbf/ft^2 a (Pa)
- h = elevation or depth, ft (m)
- γ = specific weight, lbf/ft^3 (N/m^3)

This equation states that at any point within a given fluid the pressure decreases as the elevation (z) increases, at a local rate that is equal to the product of the fluid density and the gravitational acceleration at that point[2]. Hydrostatic equation also can be applied to several special states. They cover isochoric, isothermal, and isentropic which are described as follow.

i. Isochoric

Isochoric is a state fluid in constant volume that also states that the fluid includes constant density. Commonly, isochoric uses in incompressible flow because it has definite volume. If constant density is related in eq (166), there are only pressure and elevation which include as variables at two points. Eq (166) can also written as follow.

$$p_1 + \rho \cdot g \cdot z_1 = p_2 + \rho \cdot g \cdot z_2 \qquad \text{Eq (167)}$$

This equation is called potential, Φ that states that sum of the pressure (P) and static head (ρgz) at all points is constant within a given isochoric (incompressible) fluid. This is an important result for such fluids, and it can be applied directly to determine how the pressure varies with elevation in a static liquid.

ii. Isothermal

Isothermal is happened when temperature fluid is constant for all elevation, z and by following ideal gas law. In this case, hydrostatic can be obtained by relating its equation with Eq (163), then, it becomes the pressure as a function of elevation. The pressure drops exponentially as the elevation increases instead of linearly as for the incompressible fluid. It can be appeared as below.

$$p_2 = p_1 \exp\left(- \frac{MW \cdot g \cdot (z_2 - z_1)}{RT} \right) \qquad \text{Eq (168)}$$

iii. Isentropic

Isentropic is one for which there is no heat transfer in reversible between the system and surroundings, therefore this process is also adiabatic. For an ideal gas under these conditions, the isentropic equation can be related by Eq (163) and (167). Thus, It results as given in Eq (169) for the pressure as function of elevation and Eq (170) for the temperature under isentropic equation that states that the temperature drops linearly as the elevation increases.

$$P_2 = P_1\left[1-\left(\frac{k-1}{k}\right)\left(\frac{g \cdot MW \cdot (z_2 - z_1)}{RT_1}\right)\right]^{\frac{k}{(k-1)}} \qquad \text{Eq (169)}$$

$$T_2 = T_1\left[1-\left(\frac{k-1}{k}\right)\left(\frac{g \cdot MW \cdot (z_2 - z_1)}{RT_1}\right)\right] \qquad \text{Eq (170)}$$

Where,

k = specific heat ratio for the gas, Cp/Cv

b. Pressure-Sensing Device

The principal device using liquids is manometer. Manometer measures the pressure difference between two points in a fluid (flowing or static). The manometer contains an incompressible liquid (density, ρ_m) that is immiscible with the fluid flowing in the pipe (density, ρ_f).

It can be shown in figure 43.

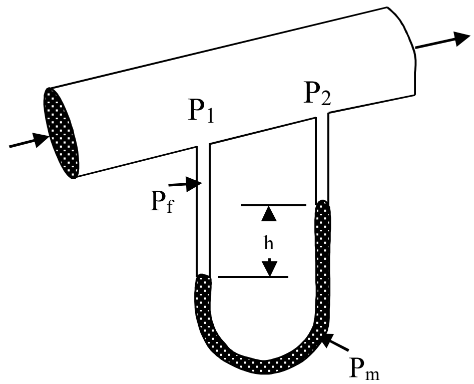


Figure 43 : manometer

Based on the figure, the equation is expressed by

$$p_1 - p_2 = (\rho_m - \rho_f)gh \qquad \text{Eq (171)}$$

c. Hydrostatic Forces on Submerged Plane Surface

The force on a submerged plane surface is equal to the pressure at the depth of its centroid multiplied by its area. The point at which the force acts is called the ‘centre of pressure’ and is at a greater depth than the ‘center of gravity’ (centroid). These forces are only relevant if one side of the surface is exposed to a pressure which does not depend on the depth (e.g. the sides of a vessel, an immersed gate or manhole, a dam wall, etc.).

They can be illustrated in figure 44.

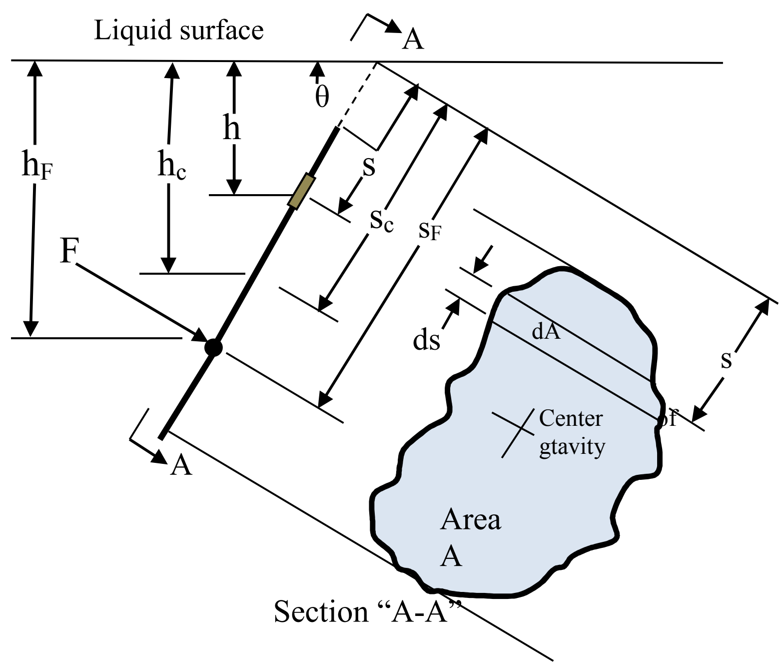


Figure 44 : notation for liquid force on submerged plane surface

The force exerted by a liquid on a plane area to center of pressure due to the hydrostatic pressure is given by

$$F = \rho \cdot g \cdot h_C \cdot A \qquad \text{Eq (172)}$$

The location of the center of this force is given by

$$s_F = s_C + \frac{I_G}{s_C A} \qquad \text{Eq (173)}$$

Based on that equation, depth of center of pressure for an angle plane can be obtained as follows.

$$h_F = h_C + \frac{I_G (\sin\theta)^2}{h_C A} \quad \text{Eq (174)}$$

Where,

- F = force exerted by a liquid on a plane area, lbf (N)
- h_C = distance from the liquid surface to the center of gravity, ft (m)
- h_F = distance from the liquid surface to the center of pressure, ft (m)
- s_F = inclined distance from the liquid surface to the center of pressure, ft (m)
- s_C = inclined distance to the center of gravity of the surface, ft (m)
- I_G = moment of inertia around its center of gravity, lbm.ft^2 (kg.m^2)
- A = area of the surface, ft^2 (m^2)

i. Hydrostatic Forces on Submerged Curved Surface

In this case, the hydrostatic force is obtained from the resultant, F_R of its horizontal, F_H and vertical, F_V components as shown in figure 45. The horizontal component on a curved surface is equal to the normal force on the vertical projection of the surface. It acts through the center of pressure for the vertical projection. It can be defined as follows[12].

$$F_H = \rho \cdot g \cdot h_F \cdot A_{AC} \quad \text{Eq (175)}$$

Whereas, the vertical component, Fv is the weight of fluid above the curve and acts through the centroid of the volume of liquid above that is expressed as follows.

$$F_V = \rho \cdot g \cdot V_{ABEF} \quad \text{Eq (176)}$$

$$F_V = \rho \cdot g \cdot (V_{ADEF} + V_{ABD}) \quad \text{Eq (177)}$$

The resultant pressure force on the curved surface, F_R is given by

$$F_R = \sqrt{F_H^2 + F_V^2} \quad \text{Eq (178)}$$

The angle of inclination of the result to the horizontal is defined by

$$\alpha = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$ Eq (179)

Where,

F_H = normal force on the vertical projection, lbf (N)
F_V = weight of fluid above the curve, lbf (N)
F_R = resultant pressure force on the curved surface, lbf (N)
A = area of curved surface, ft^2 (m^2)
α = angle of inclination, degree

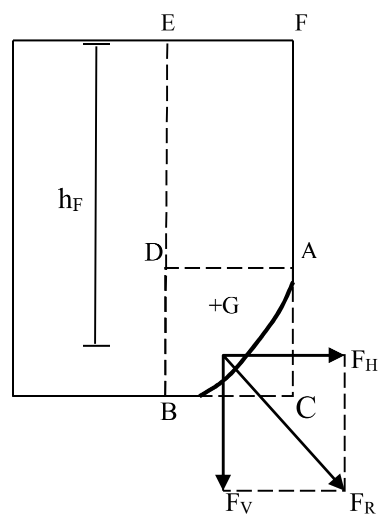


Figure 45 : notation for liquid force on submerged curved surface

d. Buoyancy

Buoyancy is based on Archimedes' principle, which states that the buoyant force exerted on a submerged body is equal to the weight of the displaced fluid. This buoyancy force acts vertically upwards through the centroid of the displaced volume, which is known as the center of buoyancy (B). If the buoyancy force is equal to the weight of the body then the body will float in the fluid. If the weight of the body is greater than the buoyancy force then the body will sink. If the buoyancy force is greater than the weight of the body then the body will rise[9].

Buoyancy force diagram can be illustrated in figure 46.

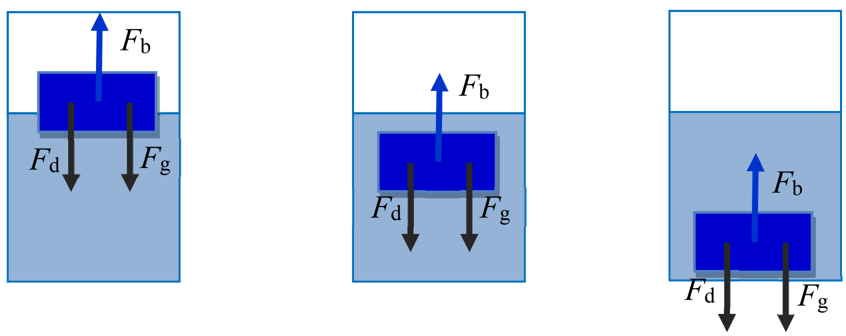


Figure 46 : buoyancy force diagram

Some buoyancy applications include determination of irregular volumes, specific gravities of liquids, and naval architectural design[2]. The buoyant force being the weight of the displaced liquid, the equilibrium equation may be defined by

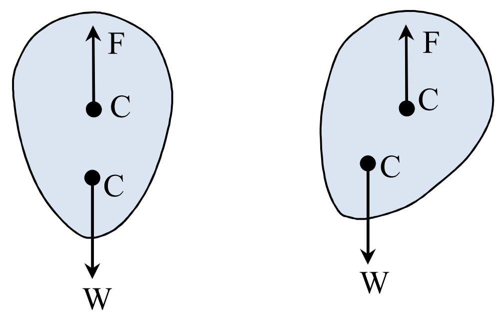
$$F_D = F_B - F_g = \gamma_f V - \gamma_b V = (\gamma_f - \gamma_b) \cdot V \qquad \text{Eq (180)}$$

Where,

F_B = buoyant force, lbf (N)
F_g = gravity force (weight of body), lbf (N)
γ_f = specific weight of the fluid, lbf/ft^3 (N/m^3)
γ_b = specific weight of the body, lbf/ft^3 (N/m^3)
V = volume of the body, ft^3 (m^3)

e. Stability of Submerged and Floating Bodies

A body is said to be in stable equilibrium when given a slight displacement from the equilibrium position, the forces thereby created tend to restore it back to its original position. In order for a submerged body to be stable, the body's center of gravity must lie below the center of buoyancy of the displaced liquid. In order for a submerged body to be in neutral equilibrium for all positions, the body's centers of gravity and buoyancy must coincide. These are illustrated in figure 47.



(a) Stable (b) Unstable

Figure 47: Stability for a submerged body

The stability problem is more complicated on whether a righting or overturning moment is developed when the centers of gravity and buoyancy move out of vertical alignment for floating bodies because as the body rotates the location of the center of buoyancy may change. The point at which the line of action of the buoyancy force, F_B cuts the vertical center line of the floating body in the displaced position, is known as the metacenter (M). For a floating body to be stable M must lie above the body's center of gravity, G. If M lies on G the body is in neutral equilibrium. The directed distance from G to M is called the metacentric height[4].

Stability of a floating body can be shown in figure 48.

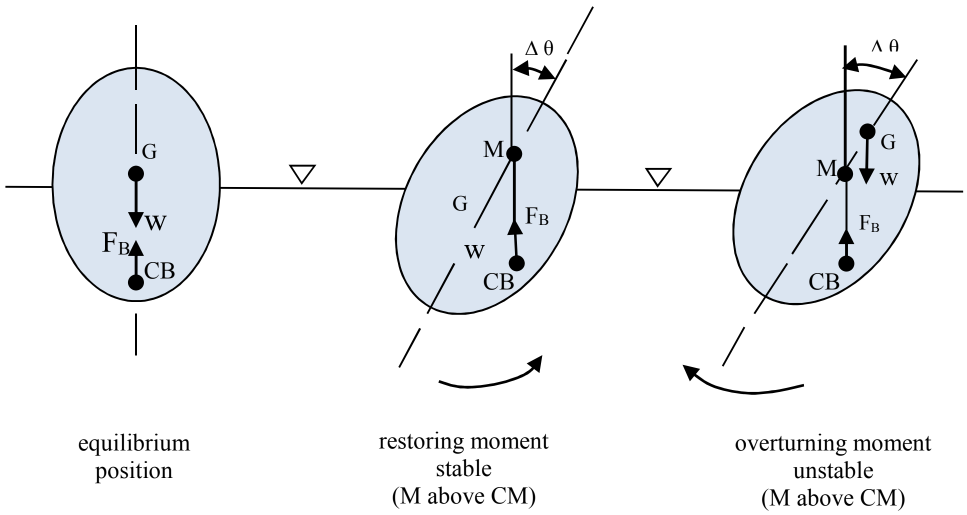


Figure 48 : Stability for a floating body

f. Compressible Flow

For gases equation, a typical case is the atmosphere, where the relationship may be taken as polytropic or isothermal, depending on the altitude. For this purpose, the geopotential altitude, H should be defined as follows[6].

$$H = \frac{Z}{\left(1 + \frac{z}{r}\right)} \qquad \text{Eq (181)}$$

For an isothermal process which the temperature remains constant for all z, pressure-altitude can be defined in Eq (182).

$$\frac{P_2}{P_1} = e^{-\frac{(H_2 - H_1)}{RT}} \qquad \text{Eq (182)}$$

For a polytropic process in $\delta \neq 1$, is particularly useful for characterizing expansion and compression processes which include heat transfer. Pressure-altitude can be defined by Eq (183). It also can be related by temperature-height as shown in Eq (184).

$$\frac{P_2}{P_1} = \left[1 - \frac{(\delta - 1)(H_2 - H_1)}{\delta R T_1}\right]^{\frac{\delta}{(\delta - 1)}} \quad \text{Eq (183)}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{(H_2 - H_1)}{R(T_1 - T_2)}} \quad \text{Eq (184)}$$

Where,

H = geopotential altitude, dimensionless
Z = compressibility factor, dimensionless
z = height above sea level, ft (m)
r = radius of the earth, 21×10^6 ft (6.4×10^6 m)
T = temperature, °R (K)
R = gas law constant, 10.731 $ft^3.lb_f/in^2$.lbmol °R (8314.34 $kg.m^2/s^2$.kgmol.K)
δ = polytropic index

III. Fluid Dynamic

a. Flow rate

Flow rate is important to measure rate at which fluid is flowing along a pipe. There are three types flow rate that is often used. They are described as follow.

a. Mass flow rate

Mass of fluid flowing through a control surface per unity time. Mass flow rate can be defined by

$$\dot{m} = \frac{m}{t} \quad \text{Eq (185)}$$

$$= \rho \cdot v \cdot A \quad \text{Eq (186)}$$

b. Volumetric flow rate

Volume of fluid flowing through a control surface per unit time. It can be defined by

$$Q = \frac{V}{t} \quad \text{Eq (187)}$$

$$= \frac{m}{\rho \times t} \quad \text{Eq (188)}$$

$$= \frac{\dot{m}}{\rho} \quad \text{Eq (189)}$$

c. Mean flow velocity

Mean velocity is velocity at area of cross section of the pipe at specific point and time. It may be defined by

$$v_m = \frac{Q}{A} \quad \text{Eq (190)}$$

Where,

$\dot{m}$ = mass flow rate, lbm/s (kg/s)
Q = volumetric flow rate, ft^3/s (m^3/s)
m = mass, lb (kg)
V = volume, ft^3 (m^3)
v_m = mean velocity, ft/s (m/s)
t = time, second or hour
A = cross sectional area, ft^2 (m^2)

b. Law of the Conservation of Mass

This law states that matter can be changed from one into another, mixtures can be separated or made, but the total amount of mass remains constant. For flow problems, this conservation law is applied to a system, which is defined as any clearly specified region or volume of fluid.

The principle is applied to fixed volumes, known as control volumes (or surfaces) as shown in figure 49.

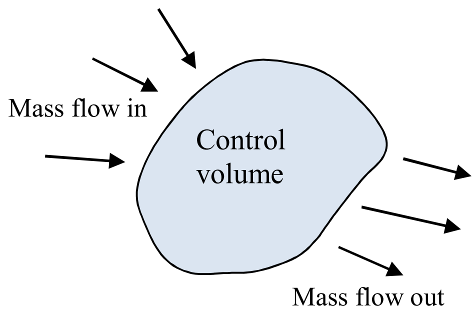


Figure 49 : An arbitrarily shaped control volume

For any control volume the principle of conservation of mass says :

Mass entering per unit time = Mass leaving per unit time

+ Accumulation of mass in the control volume per unit time   Eq (191)

Continuity equation

The equation of continuity is obtained from the principle of conservation of mass. For steady flow, the principle of conservation of mass becomes

$$\rho_1 \cdot A_1 \cdot v_{m1} = \rho_2 \cdot A_2 \cdot v_{m2} \qquad \text{Eq (192)}$$

When the fluid can be considered incompressible, i.e. the density does not change, it become volumetric flow rate as in Eq (193).

$$Q_1 = Q_2 \qquad \text{Eq (193)}$$

$$A_1 \cdot v_{m1} = A_2 \cdot v_{m2} \qquad \text{Eq (194)}$$

$$\left(\frac{\pi}{4} D_1^2\right) \cdot v_{m1} = \left(\frac{\pi}{4} D_2^2\right) \cdot v_{m2} \qquad \text{Eq (195)}$$

Continuity equation can be applied in figure 50.

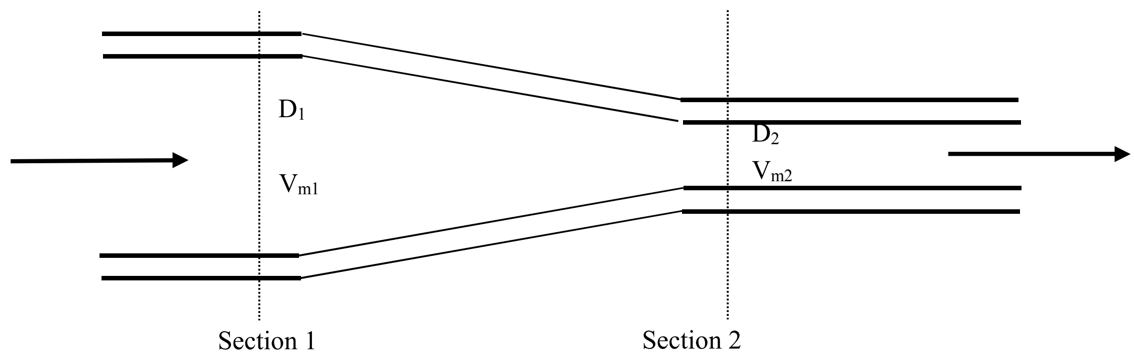


Figure 50 : continuity

c. Law of the Conservation of Energy

The principle of the conservation of energy is applied to a control volume fixed in space in much the same manner as principle of conservation of mass. The conservation of energy will then be combined with the first law of thermodynamics to obtain the final overall energy balance equation. Three forms of energy are usually considered in fluid flow problems, namely potential, kinetic, and pressure energy[8].

i. Potential energy

Potential energy (PE) is the energy possessed by an element of fluid due to its elevation above a reference datum. PE is given by

$$PE = W \cdot z = mg \cdot z \qquad \text{Eq (196)}$$

ii. Kinetic energy

Kinetic energy (KE) is the energy possessed by an element of fluid due to its velocity. The following expression can be used to calculate KE :

$$KE = \frac{1}{2\alpha} mv^2 = \frac{1}{2\alpha}\frac{Wv^2}{g}$$ Eq (197)

iii. Pressure energy

Pressure energy or flow energy (FE) is defined as the work needed to force the element of fluid across a certain distance against the pressure. The following expression applies:

$$FE = PAd = PV = P\frac{W}{\gamma}$$ Eq (198)

The total energy, E is the sum of PE, KE, and FE. It can be defined by

$$E = P\frac{W}{\gamma} + \frac{1}{2\alpha}\frac{Wv^2}{g} + W \cdot z$$ Eq (199)

Energy, E is the total energy per weight of fluid. In fluid mechanics and hydraulic problems, it is customary to work with energy expressed as a head. However, there is a difference in their units. To express the total energy (E) as a head (H), Eq. (199) can be divided by the weight of the fluid W, which gives

$$H = \frac{P}{\gamma} + \frac{1}{2\alpha}\frac{v^2}{g} + z$$ Eq (200)

By the principle of conservation of energy in steady flow, the total energy or head in the system does not change or constant. It also can be known as Bernoulli equation. It is applied between two points that illustrated in figure 51 and the equation is shown as follow.

Energy section 1 = Energy section 2 - Energy added + Energy lost
+ Energy extracted Eq (201)

$$\frac{P_1}{\gamma} + \frac{1}{2\alpha}\frac{{v_1}^2}{g} + z_1 = \frac{P_2}{\gamma} + \frac{1}{2\alpha}\frac{{v_2}^2}{g} + z_2 - h_A + h_L + h_E$$ Eq (202)

Another form of Bernoulli equation also can be defined using enthalpy definition that has involved first thermodynamics law. It is expressed by[11]

$$H_2 - H_1 + \frac{1}{2\alpha}\left(v_2^2 - v_1^2\right) + g\left(z_2 - z_1\right) = Q - W_s \qquad \text{Eq (203)}$$

For english unit,

$$H_2 - H_1 + \frac{1}{2\alpha g_c}\left(v_2^2 - v_1^2\right) + \frac{g}{g_c}\left(z_2 - z_1\right) = Q - W_s \qquad \text{Eq (204)}$$

Where,

E	= total energy per weight, btu/lb (J/kg)
H	= head, ft (m)
H_1	= enthalpy at point 1, btu/lb (J/kg)
H_2	= enthalpy at point 2, btu/lb (J/kg)
h_A	= head added, ft (m)
h_L	= head loss, ft (m)
h_E	= head extracted, ft (m)
W	= weight of the considered element, lbf (N)
v	= velocity of fluid, ft/s (m/s)
z	= distance where the element is located with respect to the datum, ft (m)
d	= distance over, ft (m)
Q	= energy added, Btu/lb (J/kg)
Ws	= net mechanical work, Btu/lb (J/kg)
∑F	= frictional losses, Btu/lb (J/kg)
g	= gravity acceleration, g_c= 32.17 ft/s^2 (9.81 m/s^2)
α	= kinetic energy velocity correction factor : 2 for laminar flow 1.02 – 1.15 for turbulent flow

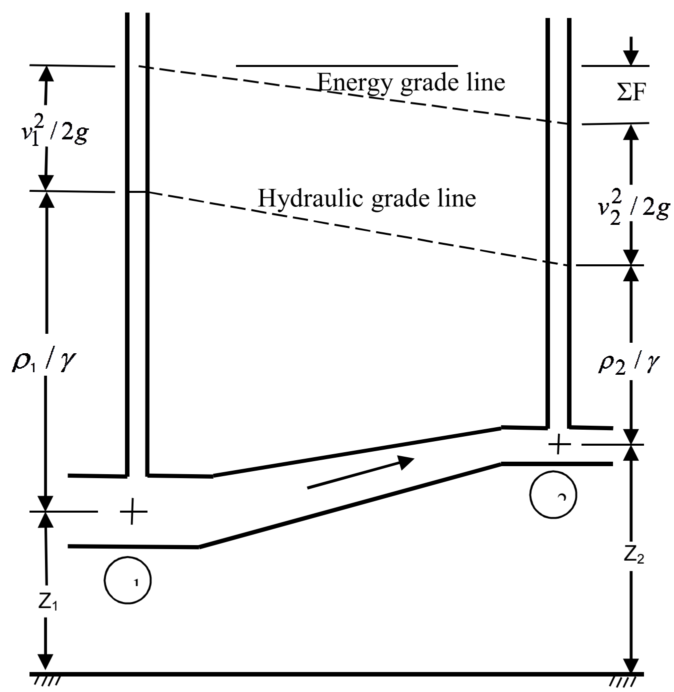


Figure 51 : process flow diagram for conservation of energy

In fact, there are several important states as fundamental to solve gasses or vapor problem in ideal gas in fluid mechanics. There are several processes which should be known. Basically, thermodynamics principles must be understood before.

d. The Power of a Stream of Fluid

If the weight per unit time of fluid flowing is known, the power of the stream can be calculated by

$$P = W_s \cdot \dot{m} \qquad \text{Eq (205)}$$

If the volume rate of flow is known,

$$P = Q \cdot \gamma \cdot H \qquad \text{Eq (206)}$$

Where,

P = Power, HP (J/s)

Ws = net mechanical work, btu/lb (J/kg)

e. Principle of Thermodynamics

Based on first law of thermodynamics, there are two forms of energy transfer, work (W) and heat (Q). They are illustrated in figure 52.

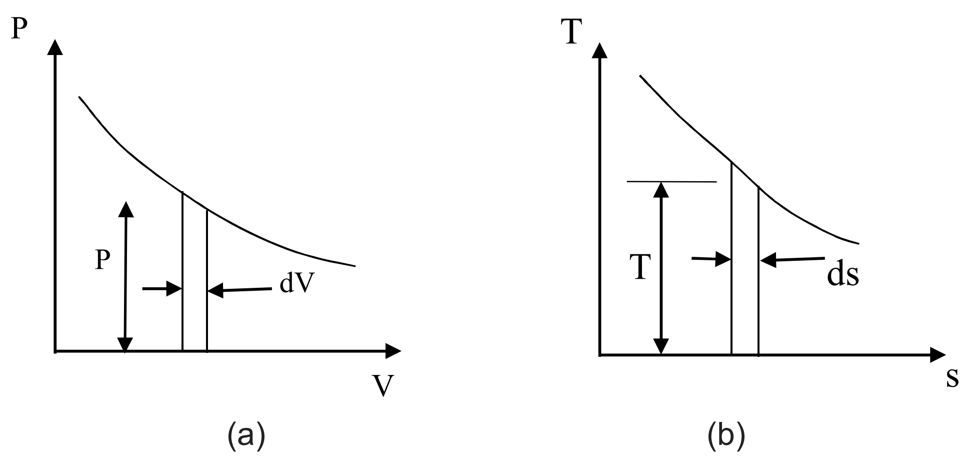


Figure 52 : P-V and T-S diagrams

Figure 52 is definition of work and heat which is also related with Eq (1). It can be expressed by

$$T\,dS - P\,dV = Cv\,dT \qquad \text{Eq (207)}$$

And then, entropy is also can be obtained on a differential basis by Eq (207). Entropy is a very important thermodynamic process that is used in the solution of gas and vapour problems.

$$dS = \frac{dQ}{T} \qquad \text{Eq (208)}$$

Where,

T	= temperature, °F (K)
S	= entropy, Btu/lbm.°F (kJ/kg.K)
P	= pressure, psia (N/m^2)
V	= volume, ft^3 (m^3)
Cv	= constant volume heat capacity, Btu/lbmol.°F (J/mol.K)

There are several thermodynamic processes in an ideal gas. The ideal gas is a model fluid described by simple property relations, which are frequently good approximations when applied to actual gases. They include isobar (constant pressure), isochoric (constant volume), isothermal (constant temperature), isentropic (adiabatic) and polytropic. They can be illustrated in figure 6 and summaried in table 3.

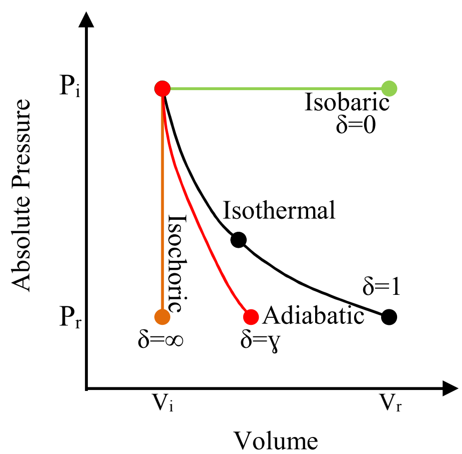


Figure 53 : thermodynamic processes in an ideal gas

Table 4: summary of processes in an ideal gas

Process	W	Q	ΔU	ΔS
Isobar	$p(V_2 - V_1)$	$\frac{pC_p}{R}(V_2 - V_1)$	$\frac{pC_v}{R}(V_2 - V_1)$	$nC_p In\left(\frac{V_2}{V_1}\right)$
Isochor	0	$\frac{VC_v}{R}(p_2 - p_1)$	$\frac{VC_v}{R}(p_2 - p_1)$	$nC_V In\left(\frac{p_2}{p_1}\right)$
Isotherm	$nRT \ln\left(\frac{V_2}{V_1}\right)$	$nRT \ln\left(\frac{V_2}{V_1}\right)$	0	$nR \ln\left(\frac{V_2}{V_1}\right)$
Isentropic	$\frac{RT_1}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$	0	$\frac{RT_1}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$	0
Polytropic	$\frac{RT_1}{\delta - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\delta-1)/\delta} - 1\right]$	0	0	$\ln \frac{P_1}{P_2}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

Specific for polytropic, it should consider its general relation which can be defined by

$$PV^{\delta} = Constant \qquad \text{Eq (209)}$$

Where,

δ = polytropic index

R = gas law constant, 10.731 $ft^3.lb_f/in^2.lbmol\ ^oR$ (8314.34 $kg.m^2/s^2.kgmol.K$)

γ = specific heat ratio, Cp/Cv

f. Conservation of Momentum

A conservation of momentum can be written for control volume shown in figure 49 which in somewhat similar to the conservation of mass. The momentum equation is a statement of Newton's Second Law and relates the sum of the forces acting on an element of fluid to its acceleration or rate of change of momentum. The equation represents in three dimensional, because momentum is a vector. Newton's second laws state that the rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force. In a steady flow situation the change related to time is zero, so the change of momentum is usually taken to be the product of the mass flow rate and the change in velocity with displacement. The rate of change of momentum can be defined by[7]

$$\sum F = \frac{d\left(\dot{m}\vec{v}\right)}{dt} \qquad \text{Eq (210)}$$

The result of integrated Eq (210),

$$\sum_{onfluid}\vec{F} = \dot{m}\left(\sum_{out}\overrightarrow{v_o} - \sum_{in}\overrightarrow{v_i}\right) \qquad \text{Eq (211)}$$

Where,

F = force, rate of change of momentum, lbf (N)
$\dot{m}$ = mass flow rate, lbm/s (kg/s)
v = velocity, ft/s (m/s)

In fact, Because momentum is vector, the force term, ∑F is composed of the sum of several forces. These force are applied for each direction. These are given as follows[11] :

1. Body force

   The body force, F_g is directed force caused by gravity acting on the total mass in the control volume. This force is mg. It is zero if the x direction is horizontal.

2. Pressure force

   The force, F_p is directed force caused by the pressure forces acting on the surface of the fluid system.

3. Friction force

   When the fluid is flowing, an directed shear or friction force, F_s is present, which is exerted on the fluid by a solid wall when the control surface cuts between the fluid and solid wall.

4. Solid surface force
   In cases where the control surface cuts through a solid, there is present force R, which is component of the resultant of the forces acting on the control volume at these points. The force of fluid on solid, ∑F is the negative of force.

The force terms can then be represented as

$$\sum F = F_g + F_p + F_s + R \qquad \text{Eq (212)}$$

i. Conservation of Momentum in One Direction

A quite common application of conservation of momentum equation is the case of a section of a conduit with its axis in the x direction as shown in figure 54.

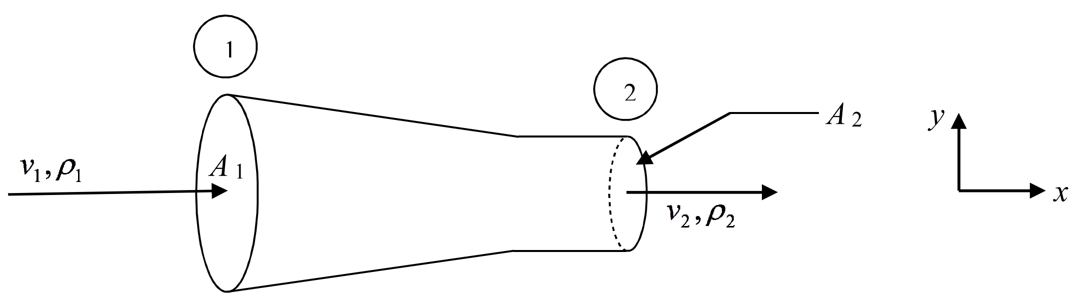


Figure 54 : a conduit in the x direction only

For the x direction, the force term in Eq (212) becomes as follows:

$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x = \dot{m}\frac{\left(v_{x2}^2\right)_m}{v_{x2m}}\cos\alpha_2 - \dot{m}\frac{\left(v_{x1}^2\right)_m}{v_{x1m}}\cos\alpha_1 \qquad \text{Eq (213)}$$

Which, $\alpha = 0^o$ in x direction, then $\cos 0^o = 1$ and

$$\frac{\left(v_x^2\right)_m}{v_{xm}} = \frac{v_{xm}}{\beta}$$ Eq (214)

Where,

F = Force, lbf (N)
R = Solid surface force or reactive force, lbf (N)
v_{xm} = mean velocity, ft/s (m/s)
α = angle, degree
β = momentum velocity correction factor
0.95 – 1 for turbulent flow
0.75 for laminar flow

In this direction, it can be analyzed as below:

- The term pressure force, F_{xp} is defined by

$$F_{xp} = p_1 A_1 - p_2 A_2$$ Eq (215)

- The friction force will be neglected, so F_{xs} = 0
- The body force F_{xg} = 0, since gravity is acting only in the y direction
- Solid surface force on the fluid can be solved by relating forces Eq (213) with Eq (214) and setting β = 1 as follow:

$$R_x = \dot{m} v_{xm2} - \dot{m} v_{xm1} + p_2 A_2 - p_1 A_1$$ Eq (216)

$\sum F = -R_x$ Eq (217)

ii. Conservation of Momentum in Two Directions

Calculation of this section is close with momentum in one direction. However, it should consider directions and angle. A case of system can be shown in figure 55.

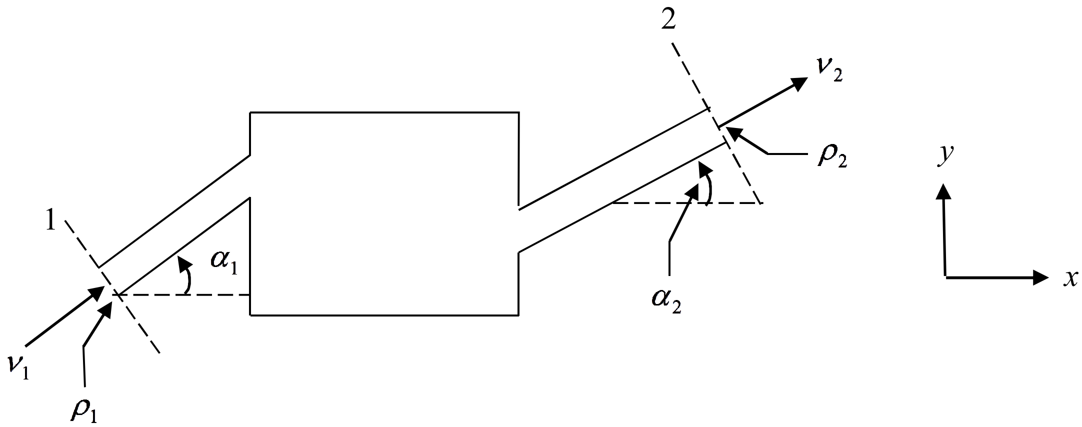


Figure 55 : fluid entering at point 1 and leaving at point 2.

In this case, Rx and Ry should be analyzed one by one. For x direction, the force equation is using Eq (213). It can be analyzed as below:

- The term pressure force, F_{xp} is defined by

$$F_{xp} = p_1 A_1 \cos \alpha_1 - p_2 A_2 \cos \alpha_2 \qquad \text{Eq (218)}$$

- The friction force will be neglected, F_{xs} = 0 and the body force F_{xg} = 0
- Solid surface force on the fluid can be solved by relating forces with Eq (213) and setting β = 1 as follow:

$$R_x = \dot{m} v_{xm2} \cos \alpha_2 - \dot{m} v_{xm1} \cos \alpha_1 + p_2 A_2 \cos \alpha_2 - p_1 A_1 \cos \alpha_1 \qquad \text{Eq (219)}$$

For R_y, the body force F_{yg} is negative y direction and F_{yg} = -mg. Replacing cos α by sin α, the equation for y direction becomes

$$R_y = \dot{m} v_{ym2} \sin \alpha_2 - \dot{m} v_{ym1} \sin \alpha_1 + p_2 A_2 \sin \alpha_2 - p_1 A_1 \sin \alpha_1 + mg \qquad \text{Eq (220)}$$

The result of forces can be expressed by

$$|R| = \sqrt{R_x^2 + R_y^2} \qquad \text{Eq (221)}$$

The force can be implemented as Eq (217). Whereas, the angle with vertical is expressed by

$$\theta = \arctan\left(\frac{R_x}{R_y}\right) \quad \text{Eq (222)}$$

h. Incompressible Pipe Flow

1. Flow Regimes

The two major flow regimes are laminar and turbulent. The stable flow observed initially was termed laminar flow, because it was observed that the fluid elements moved in smooth layers or relative to each other with no mixing. The unstable flow pattern, characterized by a high degree of mixing between the fluid elements, was termed turbulent flow. Although the transition from laminar to turbulent flow occurs rather abruptly, there is nevertheless a transition region where the flow is unstable but not thoroughly mixed[2].

Nature of flow in pipe whether is the laminar flow or turbulent flow is determined by Reynolds Number (Re). Laminar flow in a tube persists up to a point where the value of the Reynolds number is less than 2100, and turbulent flow occurs when NRe is greater than about 4000, with a transition region in between. The Reynolds number can be appreciated by

$$N_{\mathrm{Re}} = \frac{\rho \cdot v \cdot D}{\mu} \quad \text{Eq (223)}$$

Where,

ρ = fluid density, lb/ft^3 (kg/m^3)
v = fluid velocity, ft/s (m/s)
D = Diameter pipe, ft (m)
μ = viscosity, cP (Pa.s)

For the non-circular cross sections, equivalent hydraulic diameter will be replaced with D for calculation of Reynolds Number. It has to determine with the following formula:

$$D_e = 4\left[\frac{cross\ sectional\ area}{wetted\ perimeter}\right] \qquad \text{Eq (224)}$$

Where,

D_e = equivalent hydraulic diameter, ft (m)

2. Parameters for Pipe Flow

i. Pipe Losses (Friction)

For use in pipe flow problems with viscous fluids the Bernoulli equation may be adapted to include a head loss, h_L. Applied between two positions (1) and (2) in a pipe, the head equation gives:

$$\Delta\left(\frac{P}{\rho} + z + \frac{v^2}{2g}\right) = h_L \qquad \text{Eq (225)}$$

The head loss is using Dancy equation that is expressed as below for the horizontal pipe line,

$$h_L = \frac{\Delta P}{\rho g} = f\,\frac{L}{D}\cdot\frac{v^2}{2g} \qquad \text{Eq (226)}$$

Normally Dancy's friction factor, f for the laminar flow conditions is simple calculated with just function of the Reynolds number only, which can be expressed as

$$f = \frac{64}{\text{Re}} \qquad \text{Eq (227)}$$

Generally for the turbulent flow and transition flows, the friction factor is computed using the Colebrook equation.

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{D} + \frac{9.35}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{Eq (228)}$$

Where,

ε = absolute roughness, in (mm)

or the simplified formula can be written as

$$f = \frac{1}{\left(2\log\left[\left(\frac{7}{\mathrm{Re}}\right)^{0.9} + \left(\frac{3.24\varepsilon}{D}\right)\right]\right)^2} \qquad \text{Eq (229)}$$

Roughness is a factor denoting the roughness of the pipe or tube; the more rough the pipe, the larger the friction factor; the larger the friction factor, the more pressure drop. This value is taken from standard table by difference piping material. Normally the value of roughness for the 'commercial steel pipe' is 0.00018 in. The standard table is given in table 4.

Table 5 : absolute roughness of pipe

Pipe material	Roughness, ε, in (mm)
Steel, welded and seamless	0.0002 (0.061)
Ductile Iron	0.0002 (0.061)
Ductile Iron, asphalt coated	0.0004 (0.12)
Copper and Brass	0.002 (0.61)
Glass	0.000005 (0.0015)
Thermoplastics	0.000005 (0.0015)
Drawn Tubing	0.000005 (0.0015)

Relative roughness of the pipe is normally calculated from the Moody Chart, which expressed as

Relative roughness = $\frac{\varepsilon}{D}$ Eq (230)

Simplified the relative roughness is the ratio of the pipe internal roughness to internal size of diameter of pipe.

There are several causes of friction loss or head loss in piping systems. The effects are resulted by fittings, valves, sudden enlargement, and, sudden contraction. In additional, fluid properties, flow regimes, length, diameter and roughness of pipe also influence friction loss in pipe.

ii. Fitting and Valve Effect on Head Loss

In the fluid systems the effect of valves, elbows, and etc on the head loss is needed to be taken into consideration when designing. General pressure drop in the fitting expressed as formula for the laminar flow and turbulent flow.

$$h_f = \frac{\Delta P}{\rho g} = K\left(\frac{v^2}{2g}\right)$$ Eq (231)

Where,

K = resistance coefficient, dimensionless

Resistance coefficient of fittings and valve is given as follow.

$$K = f\frac{L_{eq}}{D}$$ Eq (232)

Representative resistance coefficient and equivalent length in pipe diameters (L_{eq}/D) of various valves and fittings can be given in table 5[11].

Table 6 : equivalent length in pipe diameters (L_{eq}/D) of various valves and fittings

Type of fitting or valve	resistance coefficient, K	Equivalent Length in Pipe diameter (L_{eq}/D)
Globe valves, wide open	6	300
Angle valves, wide open	2	100
Gate valves, wide open	0.17	9
3⁄4 open[3]	0.3	35
1⁄2 open	2.1	160
1⁄4 open	17	900
Swing check valves	2	100
Ball check valves	70	3500
90° standard elbow	0.75	35
45° standard elbow	0.35	17
Tee, Flow through run, flanged[3]	1	20
Tee, Flow through branch, flanged	0.2	60

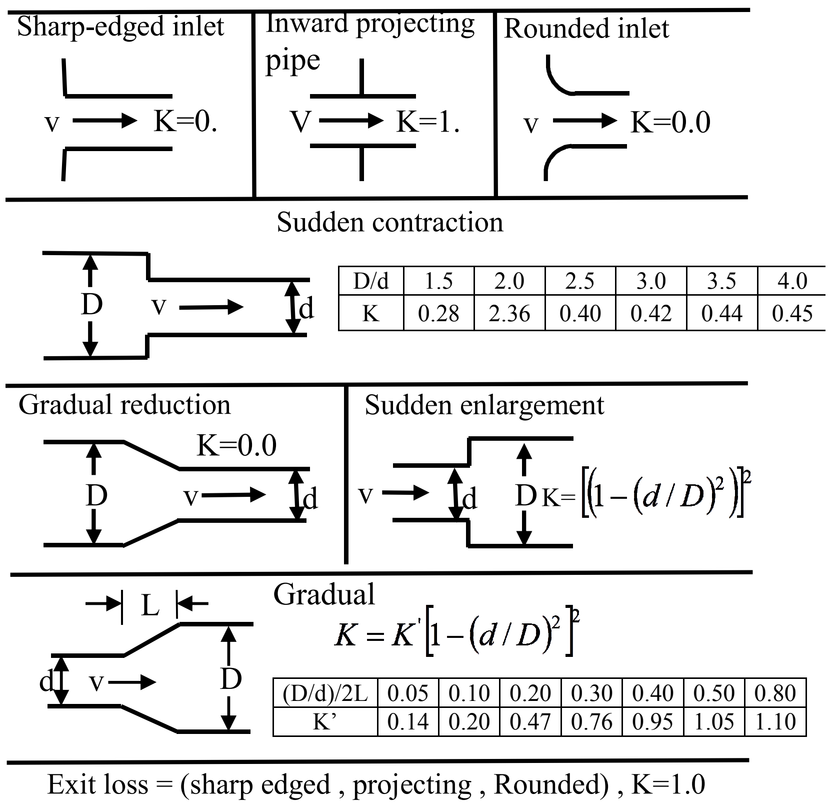
iii. Enlargements and Contraction Pipe Line Head Loss

When the fluid flow from a smaller diameter pipe goes into a bigger diameter pipe, it called enlargement, and vice-visa it called sudden contraction. Generally these processes will cause a friction loss and the changes in the kinetic energy. The friction loss is expressed as below,

$$h_e = \frac{\Delta P}{\rho g} = K\left(\frac{v^2}{2g}\right) \quad \text{Eq (233)}$$

The resistant coefficient of enlargement and contraction pipe can be expressed in table 6.

Table 7 : resistant coefficient of enlargement and contraction pipe[6]



D/d	1.5	2.0	2.5	3.0	3.5	4.0
K	0.28	2.36	0.40	0.42	0.44	0.45

(D/d)/2L	0.05	0.10	0.20	0.30	0.40	0.50	0.80
K’	0.14	0.20	0.47	0.76	0.95	1.05	1.10

i. Drag Force

When a fluid flows past a solid body or the body moves through the fluid, the force (F_D) exerted on the body by the fluid is proportional to the relative rate of momentum transported by the fluid. A number of phenomena that occur at high Reynolds number over an object describe that the free stream flow divides at the stagnation point and flows around the object. Boundary layers form on the upper and lower surfaces of the body.

The phenomena is shown in figure 56[4].

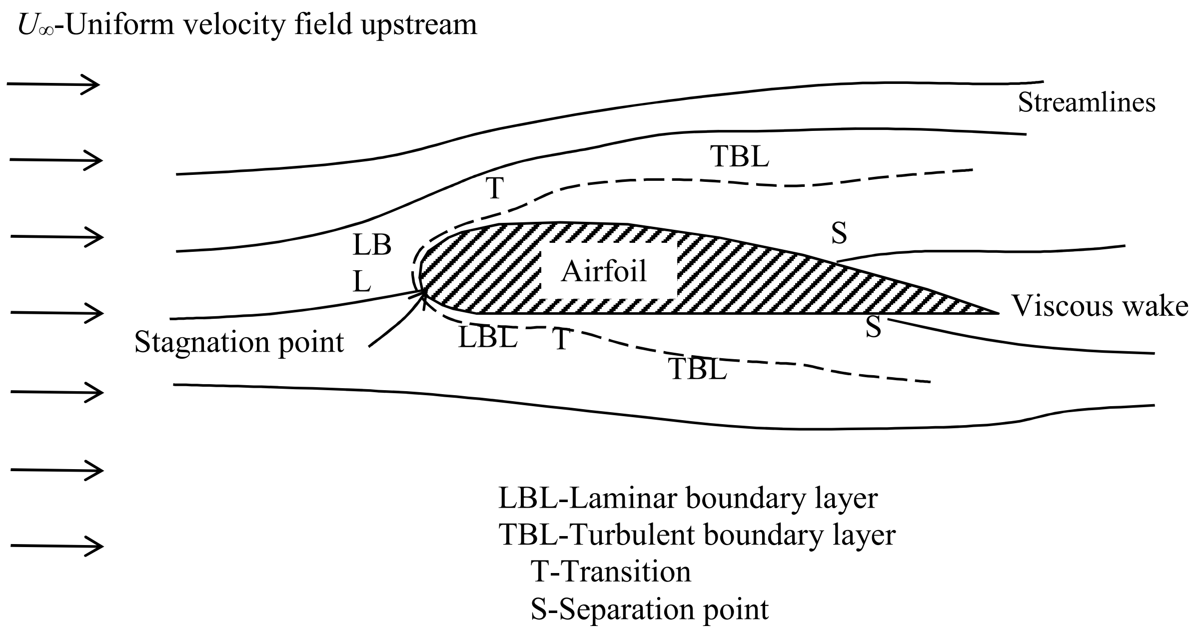


Figure 56 : viscous flow around an airfoil

Separation may occur in the region of increasing pressure on the rear of the body; after separation boundary layer fluid no longer remains in contact with the surface. Fluid that was in the boundary layers forms the viscous wake behind the object.

1. Coefficient Drag

Any object immersed in a viscous fluid flow experiences a net force from the shear stresses and pressure differences caused by the fluid motion. Drag is the force component parallel to, and lift is the force component perpendicular to, the flow direction. Drag coefficient is defined as follow.

$$C_D = \frac{F_D}{\frac{1}{2}\rho v^2 A} \qquad \text{Eq (234)}$$

Where,

C_D = drag coefficient, dimensionless
F_D = drag force, lbf (N)

However, in laminar region for low Reynold number, less than about 1.0, the drag coefficient is expressed by Stoke's law equation as follow.

$$C_D = \frac{24}{N_{Re}} \qquad \text{Eq (235)}$$

Whereas, turbulent happens for reynold number 2 x 10^5 and transition is between both. The drag coefficients can be obtained by relation of C_D versus N_{Re} exists in figure 6.

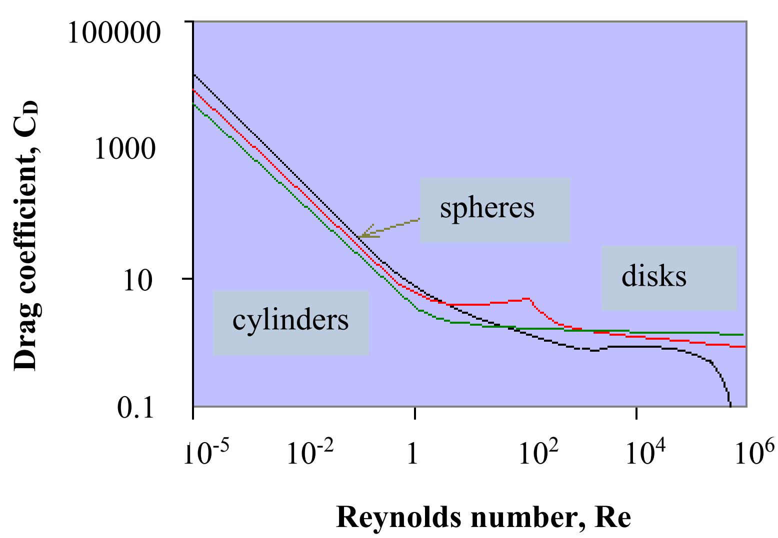


Figure 57 : drag coefficient for spheres, cylinders, and disks

REFERENCES

1. "A Textbook of Applied Mechanics" by R.K Rajput, Laxmi Publications (P) Ltd., 2011.

2. "Chemical Engineering Fluid Mechanics", second Edition, by Ron Darby, Marcel Dekker, Inc., 2001

3. "Fluid Mechanics, Fundamentals and Applications", Yunus A. Cengel, Mc Graw Hill, 2006.

4. "Fluid Mechanics - Mechanical Engineering Handbook", by Sandor Kreith, F.; Berger, S.A.; et. al., CRC Press LLC, 1999.

5. "Ludwig's Applied Process Design for Chemical and Petrochemical Plants", fourth edition, by A. Kayode Coker, Elsevier Science Ltd, 2007.

6. "Marks' Standard Handbook for Mechanical Engineers" tenth edition by Eugene A. Avallone and Theodore Baumeister III, McGraw-Hill, 1996.

7. "Mechanical Engineer's Data Handbook", by James Carvill, Elsevier Science Ltd., Inc, 2003.

8. "Mechanical Engineer's Handbook", by Dan B. Margitu, Academic Press, 2001.

9. "Mechanical Engineer's Reference Book" Twelfth edition, by Edward H. Smith, John Wiley & Sons, Inc, 2000.

10. "Solid Mechanics - Mechanical Engineering Handbook", by Sandor, B.I.; Roloff, R; et. al., CRC Press LLC, 1999.

11. "Transport Processes and Separation Process Principles" by C.J Geankoplis, Pearson Education, Inc., 2003.